

Excited collective states of nuclei within Bohr Hamiltonian with Tietz-Hua potential

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Abstract

In this paper, we present new analytical solutions of the Bohr Hamiltonian problem that we derived with the Tietz-Hua potential, here used for describing the β -part of the nuclear collective potential plus harmonic oscillator one for the γ -part. Also, we proceed to a systematic comparison of the numerical results obtained with this kind of β -potential with others which are widely used in such a framework as well as with the experiment. The calculations are carried out for energy spectra and electromagnetic transition probabilities for γ -unstable and axially symmetric deformed nuclei. In the same frame, we show the effect of the shape flatness of the β -potential beyond its minimum on transition rates calculations.

1 Introduction

The theoretical study of excited collective states in nuclei is of particular interest in nuclear structure inasmuch as it allows to understand shape phase transitions in nuclei. Therefore, different approaches have been developed in this context particularly in the framework of Bohr-Mottelson model [1, 2] and Interacting Boson Model (IBM) [3]. Moreover, the interest devoted to such a thematic has increased even more with the occurrence of critical point symmetries. The pioneer symmetries were the E(5) [4] which has been introduced to describe phase transition between vibrational and γ -unstable nuclei and the X(5) [5] one elaborated with the aim of describing the phase transition between vibrational and axially symmetric prolate deformed nuclei. Both critical point symmetries have used the infinite square well (ISW) as a recall potential for β -vibrations, while the γ -oscillations have been treated by the harmonic oscillator around $\gamma \approx 0$. Thereafter, the X(5) symmetry has generated the X(3) one [6] for γ -rigid nuclei and these two latter have been recently improved by the new X(3)-ML and X(5)-ML symmetries which have been elaborated by introducing for the first time the minimal length concept in nuclear structure [7]. Thus, the critical point symmetries have generally paved

the way for the construction of other models by making use of different potentials leading to new exactly separable models allowing the description of nuclei which are near or far from the above mentioned critical point symmetries. One can cite for example the ES-E(5) and the ES-X(5) models. The most popular among the used model potentials one can find Morse [8, 9, 10], Kratzer [11, 12], Davidson [13, 14, 15, 16], Sextic [17, 18, 19, 20, 21, 22, 23], Hulthén [24], Woods-Saxon [25], and Manning-Rosen [26, 27] potentials. Recently, it has been shown that the Morse potential [8] is more appropriate than the Davidson one in the limit of E(5) and X(5) symmetries due to its shape which becomes flat when the variable β increases in respect to the Davidson potential which grows as β^2 . As it is known, the Morse potential has been introduced for the first time in molecular physics to describe the vibrational behavior of diatomic molecules. Furthermore, different forms of such a potential have been developed like for example Tietz-Hua potential [28, 29]. This latter is considered as more realistic than the Morse one in the description of molecular dynamics as moderate and high rotational and vibrational quantum numbers [30]. Thus, in the present work, we adopted this potential within the Bohr Hamiltonian for the β -part of the separable nuclear collective potential which is chosen in the following form : $v(\beta, \gamma) = u(\beta) + w(\gamma)/\beta^2$, while the γ -part $u(\gamma)$ of this latter is taken to be equal to a harmonic oscillator potential. The elaborated model in this work has been applied to study γ -unstable and axially symmetric prolate deformed nuclei through their energy spectra and transition probabilities. These nuclear characteristics have been obtained in closed analytical form by means of Nikiforov-Uvarov method [31]. The obtained numerical results in this work are in an excellent agreement with the experimental data and fairly better than those obtained with other potentials particularly for transition rates.

The paper is organized as follows. In the Section 2, the analytical expressions for the energy levels and excited-state wave functions are presented in the γ -unstable and prolate axial rotor case, while the $B(E2)$ transition probabilities are given in Section 3. The numerical results for energy spectra and $B(E2)$ are presented, discussed, and compared with experimental data and available other models in Section 4. Finally, Section 5 contains our conclusion.

2 Theory of the model

The original collective Bohr Hamiltonian is [2]

$$H = -\frac{\hbar^2}{2B} \left[\frac{1}{\beta^4} \frac{\partial}{\partial \beta} \beta^4 \frac{\partial}{\partial \beta} + \frac{1}{\beta^2 \sin 3\gamma} \frac{\partial}{\partial \gamma} \sin 3\gamma \frac{\partial}{\partial \gamma} - \frac{1}{4\beta^2} \sum_{k=1,2,3} \frac{Q_k^2}{\sin^2(\gamma - \frac{2}{3}\pi k)} \right] + V(\beta, \gamma), \quad (1)$$

where β and γ are the usual collective coordinates, Q_k are the components of angular momentum in the intrinsic frame, and B is the mass parameter.

2.1 γ -unstable case

In order to achieve exact separation of the variables β and γ in eq. (1), we choose the total wave function in the form [32]

$$\Psi(\beta, \gamma, \theta_i) = \xi(\beta) \Phi(\gamma, \theta_i) \quad (2)$$

where $\theta_i (i = 1, 2, 3)$ are the Euler angles, and we assume the potential to be γ independent, $V(\beta, \gamma) = U_1(\beta)$.

Then, separation of variables leads to two equations: one depending only on the β variable and the other depending on the γ variable and the Euler angles,

$$\left[-\frac{1}{\beta^4} \frac{\partial}{\partial \beta} \beta^4 \frac{\partial}{\partial \beta} + \frac{\Lambda}{\beta^2} + u_1(\beta) \right] \xi(\beta) = \epsilon \xi(\beta) \quad (3)$$

and

$$\left[-\frac{1}{\sin 3\gamma} \frac{\partial}{\partial \gamma} \sin 3\gamma \frac{\partial}{\partial \gamma} + \frac{1}{4} \sum_k \frac{Q_k^2}{\sin^2(\gamma - \frac{2}{3}\pi k)} \right] \Phi(\gamma, \theta_i) = \Lambda \Phi(\gamma, \theta_i), \quad (4)$$

where Λ is the separation constant and the following notations are used $u_1(\beta) = \frac{2B}{\hbar^2} U_1(\beta)$ and $\epsilon = \frac{2B}{\hbar^2} E$.

The γ and Euler angles equation (4) has been solved by Bès[33]. In this equation, the eigenvalues of the second-order Casimir operator $\text{SO}(5)$ are expressed in the following form $\Lambda = \tau(\tau+3)$, where τ is the seniority quantum number, characterizing the irreducible representations of $\text{SO}(5)$ and taking the values $\tau = 0, 1, 2, \dots$ [34].

The values of angular momentum L occurring for each τ are provided by a well known algorithm and are listed in [3]. The ground state band levels are determined by $L = 2\tau$.

2.2 The prolate axial rotor case

In this case, the reduced potential $v(\beta, \gamma) = 2BV/\hbar^2$ depends on the asymmetry γ . However, to achieve exact separation of variables [35, 12, 36, 9, 32], we assume a potential of the form $v(\beta, \gamma) = u_1(\beta) + u_2(\gamma)/\beta^2$, given in Eq. (1).

For the γ -potential, we use a harmonic oscillator [5, 36]

$$u_2(\gamma) = (3c)^2 \gamma^2 \quad (5)$$

where c is a free parameter.

As the γ potential is minimal at $\gamma = 0$, one can write the angular momentum term of Eq. (1) as [5]

$$\sum_{k=1,2,3} \frac{Q_k^2}{\sin^2(\gamma - \frac{2}{3}\pi k)} \approx \frac{4}{3}(Q_1^2 + Q_2^2 + Q_3^2) + Q_3^2 \left(\frac{1}{\sin^2 \gamma} - \frac{4}{3} \right) \quad (6)$$

Using wave functions of the form

$$\Psi(\beta, \gamma, \theta_i) = F_L(\beta) \eta_K(\gamma) \mathcal{D}_{M,K}^L(\theta_i) \quad (7)$$

where $\mathcal{D}(\theta_i)$ denote Wigner functions of the Euler angles. L are the eigenvalues of angular momentum, while M and K are the eigenvalues of the projections of angular momentum on the laboratory fixed x -axis and the body-fixed x' -axis respectively. The separation of variables leads to

$$\left[-\frac{1}{\beta^4} \frac{\partial}{\partial \beta} \beta^4 \frac{\partial}{\partial \beta} + \frac{\bar{\Lambda}}{\beta^2} + u_1(\beta) \right] \xi(\beta) = \epsilon \xi(\beta) \quad (8)$$

$$\left[-\frac{1}{\sin 3\gamma} \frac{\partial}{\partial \gamma} \sin 3\gamma \frac{\partial}{\partial \gamma} + \frac{K^2}{4} \left(\frac{1}{\sin^2 \gamma} - \frac{4}{3} \right) + u_2(\gamma) \right] \eta_K(\gamma) = \hat{\Lambda} \eta_K(\gamma) \quad (9)$$

where $\bar{\Lambda} = \hat{\Lambda} + L(L+1)/3$ and $\hat{\Lambda}$ is a parameter coming from the exact separation of the variables, obtained from the γ equation. The equation (9) has been solved [5] for the potential (5) leading to

$$\hat{\Lambda} = (6c)(n_\gamma + 1) - \frac{K^2}{3} \quad (10)$$

where n_γ is the quantum number related to γ -oscillations.

2.3 Common form of the radial part

We see that Eq. (3) has the same form as (8), obtained in the γ -unstable case, the only difference being that Λ in the first equation is replaced by $\bar{\Lambda}$ in the axially symmetric prolate deformed nuclei. In what follows we are going to use the symbol Λ .

In this work, we use the Tietz-Hua potential with a unit depth expressed as [28, 29]

$$u_1(\beta) = \left[\frac{1 - e^{-b_h(\beta - \beta_e)}}{1 - c_h e^{-b_h(\beta - \beta_e)}} \right]^2 \quad (11)$$

By inserting the function $f(\beta) = \beta^{-2}\xi(\beta)$ in the radial equation (3), we obtain:

$$\left[-\frac{d^2}{d^2\beta} + \frac{\Lambda + 2}{\beta^2} + \left[\frac{1 - e^{-b_h(\beta - \beta_e)}}{1 - c_h e^{-b_h(\beta - \beta_e)}} \right]^2 \right] f(\beta) = \epsilon f(\beta). \quad (12)$$

For a small β deformation, the centrifugal potential could be expanded around $\beta = \beta_e$ in a series of powers of $x = (\beta - \beta_e)/\beta_e$ as

$$V_l(\beta) = \frac{\Lambda + 2}{\beta^2} = \frac{\Lambda + 2}{\beta_e^2(1+x)^2} = \frac{\Lambda + 2}{\beta_e^2}(1 - 2x + 3x^2 - 4x^3 + \dots) \quad (13)$$

It is sufficient to keep expansion terms only up to the second order. The following form of the potential can be used instead of the centrifugal potential in the Pekeris approximation [37]

$$\tilde{V}_l(\beta) = \frac{\Lambda + 2}{\beta_e^2} \left(D_0 + D_1 \frac{e^{-\alpha x}}{1 - c_h e^{-\alpha x}} + D_2 \frac{e^{-2\alpha x}}{(1 - c_h e^{-\alpha x})^2} \right) \quad (14)$$

where $\alpha = b_h\beta_e$ and D_i is the parameter of coefficients ($i = 0, 1, 2$). By expanding Eq. (14) up to terms x^2 , after making some arrangements and combining equal power with Eq. (13), we obtain the relations between the coefficients and parameters α and c_h as follows:

$$D_0 = 1 - \frac{1}{\alpha}(1 - c_h)(3 + c_h) + \frac{3}{\alpha^2}(1 - c_h)^2, \quad (15)$$

$$\lim_{c_h \rightarrow 0} D_0 = 1 - \frac{3}{\alpha} + \frac{3}{\alpha^2},$$

$$D_1 = \frac{2}{\alpha}(1 - c_h)^2(2 + c_h) - \frac{6}{\alpha^2}(1 - c_h)^3, \quad (16)$$

$$\lim_{c_h \rightarrow 0} D_1 = \frac{4}{\alpha} + \frac{6}{\alpha^2},$$

$$D_2 = -\frac{1}{\alpha}(1 - c_h)^3(1 + c_h) + \frac{3}{\alpha^2}(1 - c_h)^4, \quad (17)$$

$$\lim_{c_h \rightarrow 0} D_2 = -\frac{1}{\alpha} + \frac{3}{\alpha^2},$$

Rewriting Eq. (12) by using the new variable $z = c_h e^{-\alpha x}$, we obtain

$$\left[\frac{d^2}{dz^2} + \frac{1-z}{z(1-z)} \frac{d}{dz} + \left(\frac{\beta_e^2 \epsilon}{\alpha^2} (1-z)^2 - \frac{\beta_e^2}{\alpha^2} \left(1 - \frac{1}{c_h} z \right)^2 - \frac{\Lambda+2}{\alpha^2} \left(D_0 + \frac{D_1}{c_h} z(1-z) + \frac{D_2}{c_h^2} z^2 \right) \right) \right] f(z) = 0 \quad (18)$$

Using the generalized Nikiforov-Uvarov method [31], we obtain the energy spectrum of the β part as

$$\epsilon_{n,\Lambda} = \frac{\Lambda+2}{\beta_e^2} D_0 \quad (19)$$

$$- \frac{\alpha^2}{4\beta_e^2} \left[\frac{\frac{(\Lambda+2)}{\alpha^2 c_h^2} (D_2 - c_h D_1) + \frac{\beta_e^2}{\alpha^2 c_h^2} (1 - c_h^2)}{n + \frac{1}{2} + \sqrt{\frac{\Lambda+2}{\alpha^2 c_h^2} D_2 + \frac{\beta_e^2}{\alpha^2 c_h^2} (1 - c_h)^2 + \frac{1}{4}}} - \left(n + \frac{1}{2} + \sqrt{\frac{\Lambda+2}{\alpha^2 c_h^2} D_2 + \frac{\beta_e^2}{\alpha^2 c_h^2} (1 - c_h)^2 + \frac{1}{4}} \right) \right]^2$$

In the following, we obtained the corresponding wave function of the β part as

$$\psi(t) = N_n (1-t)^\mu (1+t)^\nu P_n(2\mu, 2\nu-1)(t) \quad (20)$$

where $t = 1 - 2z$, $\mu = \sqrt{\frac{\Lambda+2}{\alpha^2} D_0 + \frac{\beta_e^2}{\alpha^2} D - \frac{\beta_e^2}{\alpha^2} \epsilon}$ and $\nu = 1 + \sqrt{\frac{\Lambda+2}{\alpha^2 c_h^2} D_2 + \frac{\beta_e^2}{\alpha^2 c_h^2} D(1 - c_h)^2 + \frac{1}{4}}$, in which Λ for γ -unstable nuclei it is given by $\Lambda = \tau(\tau + 3)$ while for axially symmetric prolate deformed nuclei it should be replaced by $\bar{\Lambda} = (6c)(n_\gamma + 1) + \frac{L(L+1)-K^2}{3}$.

N_n is computed via the orthogonality relation of Jacobi polynomials:

$$N_n = \left(\frac{\nu + n}{2\mu(\mu + \nu + n)} \right)^{-\frac{1}{2}} \left(\frac{(\Gamma(2\mu + 1)\Gamma(n + 1))^2}{\Gamma(2\mu + n + 1)} \frac{\Gamma(2\nu + n)}{n!\Gamma(2\nu + 2\mu + n)} \right)^{-\frac{1}{2}} \quad (21)$$

Note that to compute this constant we required the following parameterization $c_h = e^{-\alpha}$.

3 $B(E2)$ transition rates

The $B(E2)$ transition rates from an initial to a final state are given by [38]

$$B(E2; s_i, L_i \rightarrow s_f, L_f) = \frac{5}{16\pi} \frac{|\langle s_f, L_f || T^{(E2)} || s_i, L_i \rangle|^2}{(2L_i + 1)}$$

$$= \frac{2L_f + 1}{2L_i + 1} B(E2; s_f, L_f \rightarrow s_i, L_i) \quad (22)$$

where s denotes quantum numbers other than the angular momentum L .

3.1 $B(E2)$ s for γ -unstable nuclei

In the general case the quadrupole operator is defined as [32]

$$T_M^{(E2)} = t\alpha_2 = t\beta \left[\mathcal{D}_{M,0}^{(2)}(\theta_i) \cos \gamma + \frac{1}{\sqrt{2}} \left(\mathcal{D}_{M,2}^{(2)}(\theta_i) + \mathcal{D}_{M,-2}^{(2)}(\theta_i) \right) \sin \gamma \right] \quad (23)$$

where $\mathcal{D}(\theta_i)$ denotes the Wigner functions of Euler angles and t is a scale factor. Then, the full symmetrized wave function is given by

$$\Psi(\beta, \gamma, \theta_i) = \beta^{-2} \chi_{n,\tau}(\beta) \Phi_\tau(\gamma, \theta_i) \quad (24)$$

The radial function $\chi(\beta)$ is given by Eq. (20), while the angular functions $\Phi_\tau(\gamma, \theta_i)$ have the form [33]

$$\Phi_\tau(\gamma, \theta_i) = \frac{1}{4\pi} \sqrt{\frac{(2\tau+3)!!}{\tau!}} \left(\frac{\alpha_2}{\beta^2} \right)^\tau \quad (25)$$

where α_2 is defined in Eq. (23). From Eqs. (22) and (25) one obtains [39]

$$B(E2; L_{n,\tau} \rightarrow (L+2)_{n',\tau+1}) = \frac{(\tau+1)(4\tau+5)}{(2\tau+5)(4\tau+1)} t^2 I_{n',\tau+1;n,\tau}^2 \quad (26)$$

with

$$I_{n',\tau+1;n,\tau} = \int_0^\infty \beta \xi_{n',\tau+1}(\beta) \xi_{n,\tau}(\beta) \beta^4 d\beta \quad (27)$$

The τ dependence of the radial wave functions $R_n(\beta)$ of Eq. (20) is contained in μ and ν , which in turn to contain $\Lambda = \tau(\tau+3)$

3.2 $B(E2)$ s for axially symmetric prolate deformed nuclei

The $B(E2)$ transition rates for axially deformed nuclei around $\gamma = 0$ read [40]

$$B(E2; nLn_\gamma K \rightarrow n'L'n'_\gamma K') = \frac{5}{16\pi} t^2 \langle L, K, 2, K' - K | L', K' \rangle^2 I_{n,L;n',L'}^2 C_{n_\gamma,K;n'_\gamma,K'}^2 \quad (28)$$

where

$$I_{n,L;n',L'} = \int_0^\infty \beta \xi_{n',L'}(\beta) \xi_{n,L}(\beta) \beta^4 d\beta \quad (29)$$

is the integral over β , while $C_{n_\gamma K, n'_\gamma K'}$ contains the integral over γ . For $\Delta K = 0$ corresponding to transitions ($g.s. \rightarrow g.s., \gamma \rightarrow \gamma, \beta \rightarrow \beta$ and $\beta \rightarrow g.s.$), the γ -integral part reduces to the orthonormality condition of the γ -wave functions: $C_{n_\gamma K, n'_\gamma K'} = \delta_{n_\gamma, n'_\gamma} \delta_{K, K'}$. While for $\Delta K = 2$ corresponding to transitions ($\gamma \rightarrow g.s., \gamma \rightarrow \beta$), this integral takes the form.

$$C_{n_\gamma K, n'_\gamma K'} = \int \sin \gamma \eta_{n_\gamma K} \eta_{n'_\gamma K'} | \sin 3\gamma | d\gamma \quad (30)$$

In the next sections, all values of $B(E2)$ are calculated in units of $B(E2; 2_{+1} \rightarrow 0_1^+)$.

4 results

The theoretical predictions for the energy spectra of the $g.s.$, β and γ bands for γ -unstable and axially symmetric deformed nuclei are obtained from equation (19) by fitting the model parameters on the experimental data using the quality measure :

$$\sigma = \sqrt{\frac{\sum_{i=1}^N (E_i(Exp) - E_i(th))^2}{(N-1)E(2_1^+)}} \quad (31)$$

where N denotes the number of the states, while $E_i(exp)$ and $E_i(th)$ represent the experimental and theoretical energies of the i^{th} level, respectively. $E(2_1^+)$ is the energy of the first excited level of the $g.s.$ band.

In Table 1 and Table 2, we present the obtained numerical results for the $g.s.$ bandhead $R_{4/2} = E(4_g^+)/E(2_g^+)$ ratios as well as those of the β and γ bandheads, normalized to the 2_g^+ level, namely : $R_{0/2} = E(0_\beta^+)/E(2_g^+)$ and $R_{2/2} = E(2_\gamma^+)/E(2_g^+)$ respectively for γ -unstable (Table 1) and axially symmetric prolate nuclei (Table 2).

The calculations have been carried out for several nuclei from ^{98}Ru to ^{200}Pt in the γ -unstable case and from ^{150}Na to ^{250}Cf in the axially symmetric prolate one.

From Table (1), one can see that over 54 studied γ -unstable nuclei, 87% are well reproduced with $\sigma < 1$ in respect to the experimental data. While from Table (2), one can observe that only 50 % of the 59 studied axially symmetric nuclei have $\sigma < 1$. This is due to the observed discrepancy in the β -band in respect to the experiment. So, at this stage, the Tietz-Hua potential seems to be appropriate for predictions of energy spectra of γ -unstable nuclei. But, its ability to reproduce well the experimental spectra for all kind nuclei could be improved within the deformation dependent mass formalism (DDMF) [41] as it has been done with Davidson potential in Ref. [13]. The DDMF is well known to have an effect on the enhancement of the calculations precision of the energy levels of nuclei [12, 13, 16].

In Figures (1)-(2), are presented the spectra of ^{162}Yb and ^{166}Er compared with the experimental levels [42]. From these figures, we can see the overall agreement between our results and the experiment. In these figures, we also show some transition rates which are very well reproduced in comparison with the experimental data.

The exhaustive results for transition rates of ground-ground, β -ground and γ -ground transitions are given in Table (3) for several γ -unstable nuclei and in Table (4) for several axially symmetric prolate ones. Also, in both tables, are listed the obtained results by Manning-Rosen $V_{MR}(\beta)$ [27], Kratzer $V_K(\beta)$ [12], Davidson $V_D(\beta)$ [13] and Morse $V_M(\beta)$ [8] potentials for comparison.

Statistics for such a comparison are presented in Figure (4) where one can clearly see that over the 34 studied γ -unstable nuclei and 38 axially symmetric prolate ones, the larger part of these nuclei is well reproduced with $\sigma < 1$ by Tietz-Hua potential (large Pie charts). In the second rank comes Manning-Rosen potential [27] which should be naturally followed by Morse [8] as can be seen in the comparison between potentials without including the Tietz-Hua one (small Pie charts). Such a fact has already been proved in [27]. But, when comparing all potentials including Tietz-Hua, we have to notice that this latter appropriated the majority of nuclei which were well reproduced by Morse as can be seen from Tables (3)-(4). While Davidson [13] and Kratzer [12] potentials occupy the last places respectively.

The efficiency of Tietz-Hua potential in reproducing the experimental data for transitions rates in comparison with the other model potentials is due to its shape particularly for β values beyond the potential minimum. Indeed, it has already been shown [27] that while the

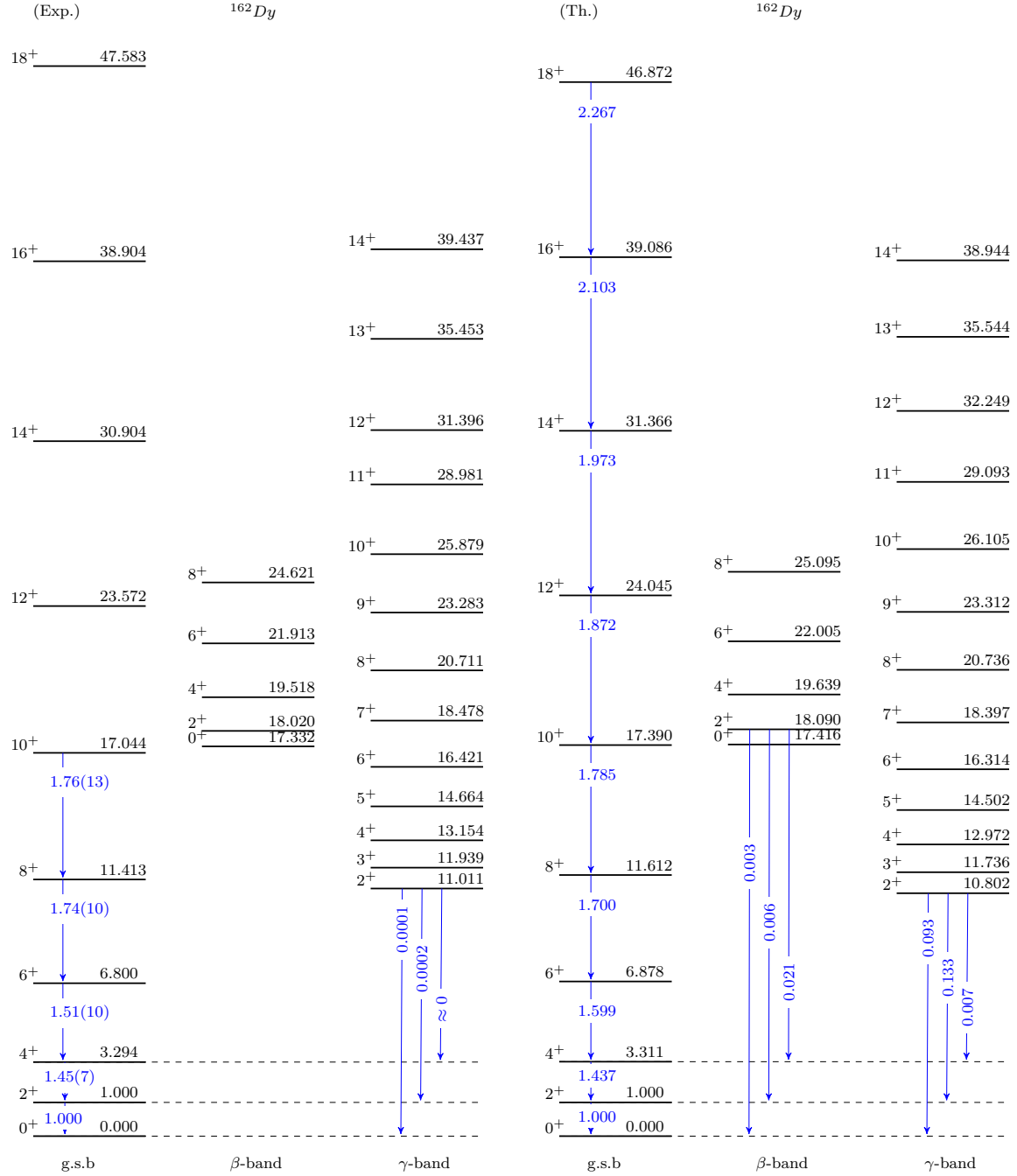


Figure 1: The theoretical energy spectra and some $B(E2)$ transitions for the ground (g.s.), γ and β bands, are compared with the experimental data [42] for ^{162}Dy .

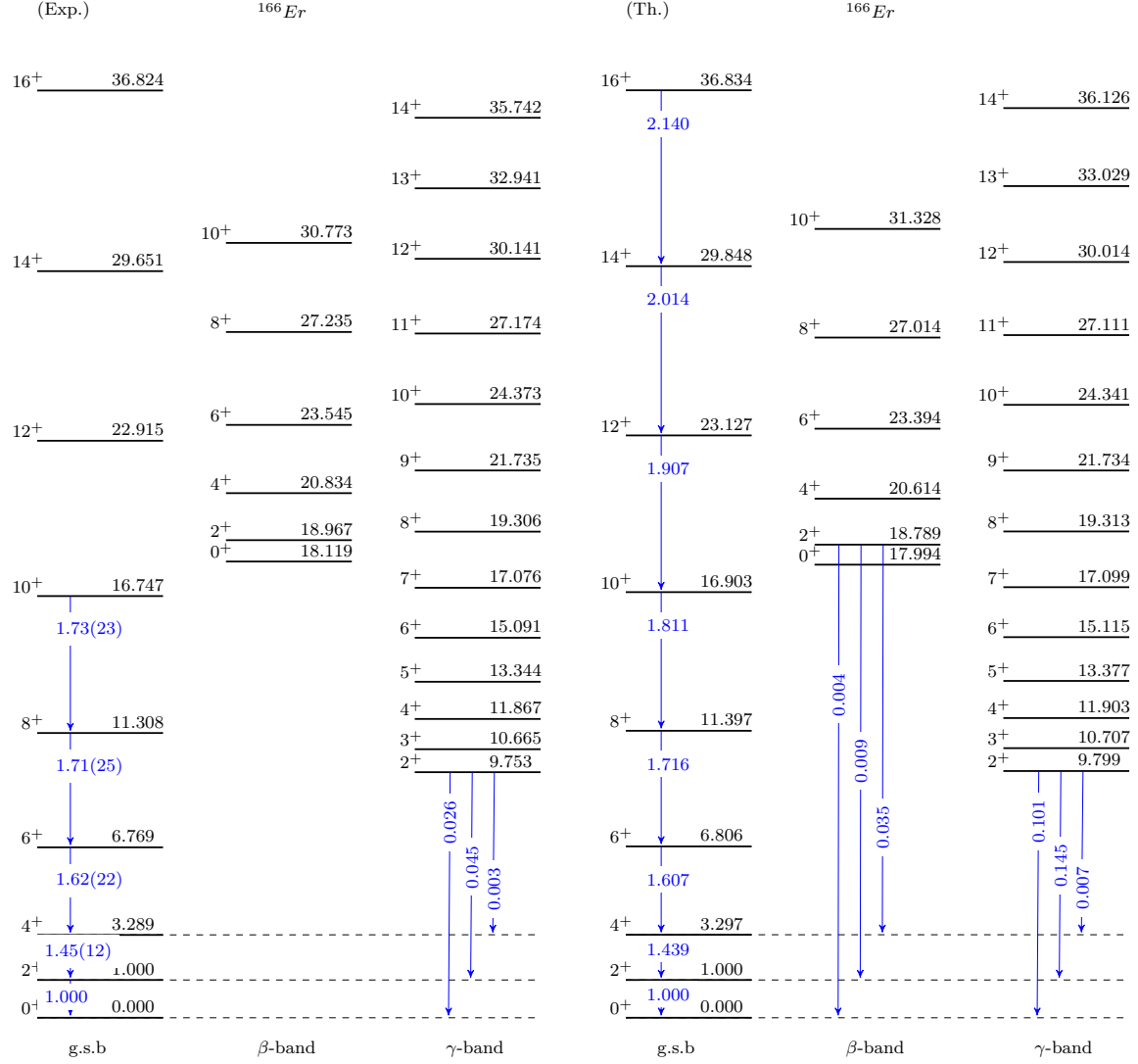


Figure 2: The theoretical energy spectra and some $B(E2)$ transitions for the ground (g.s.), γ and β bands, are compared with the experimental data [42] for ^{166}Er .

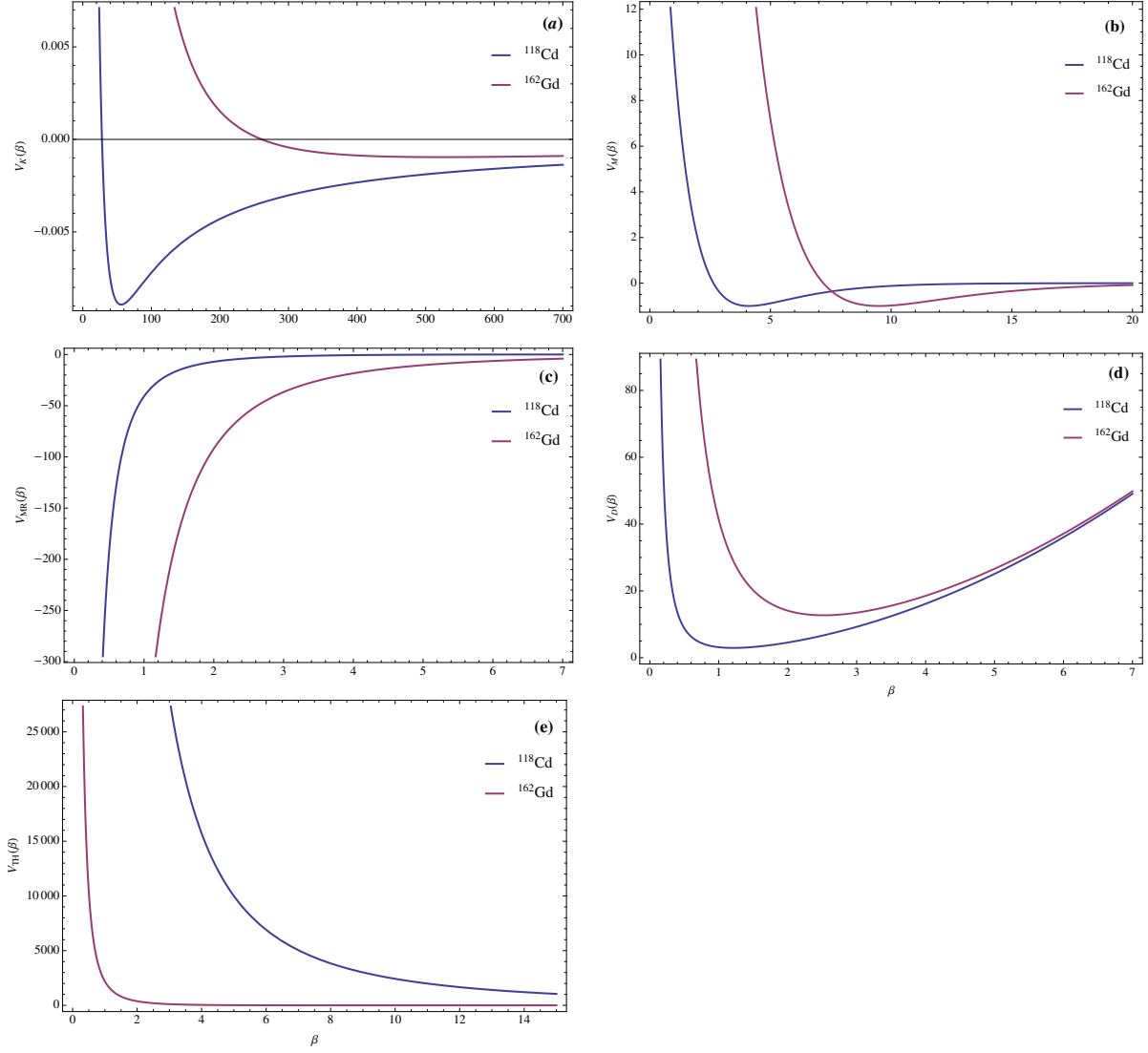


Figure 3: (Color online) Evolution of Kratzer $V_K(\beta)$ (a), Morse $V_M(\beta)$ (b), Manning-Rosen $V_{MR}(\beta)$ (c), Davidson $V_D(\beta)$ (d) and Tietz-Hua $V_{TH}(\beta)$ (e) potentials, for ^{118}Cd and ^{162}Gd nuclei. The quantities shown are dimensionless.

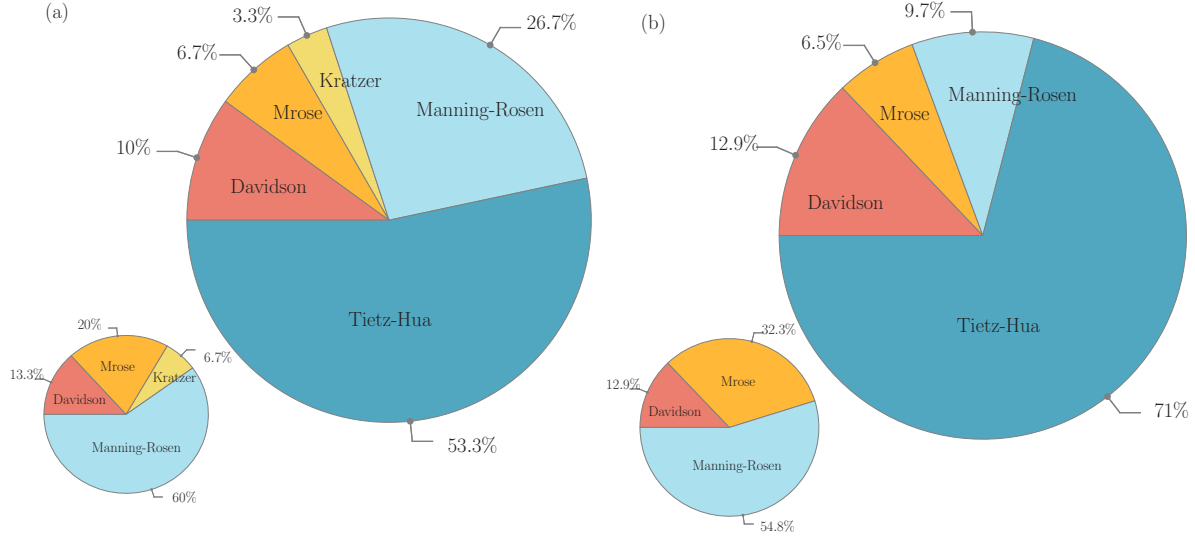


Figure 4: (Color online) The percentage of the best value of the rms in Tables (3-4) obtained by Tietz-Hua $V_{TH}(\beta)$, Manning-Rosen $V_{MR}(\beta)$ [27], Kratzer $V_K(\beta)$ [12], Davidson $V_D(\beta)$ [13] and Morse $V_M(\beta)$ [8] potentials, in the γ -unstable case (left) and the rotational case (right). The small Pie charts correspond to the statistical study without considering the Tietz-Hua potential.

considered potential is flatter beyond its minimum, the precision of calculated rates is greater. So, from Figure (3), one can see that the Tietz-Hua potential is the best candidate for this purpose.

Another characteristic, for axially symmetric prolate deformed nuclei, which has to be checked, is obviously the energy level degeneracy appearing between the β and γ bandheads, namely 0^+_{β} and 2^+_{γ} states. This property has been first suggested in the work of Alhassid-Whelan arc of regularity [43], such an arc moving inside the symmetry triangle of the IBA model [3], connecting the U(5) and SU(3) vertices. An experimental confirmation was realized [44] to identify the atomic nuclei located close to the regular region of the Casten triangle [45, 46] noted by Alhassid and Whelan. Among the studied nuclei in Table (2), the possible candidates satisfying the theoretical signature $|E(2^+_{\gamma}) - E(0^+_{\beta})|/E(2^+_{\gamma}) \leq 0.05$ [43] and the experimental one $|E(2^+_{\gamma}) - E(0^+_{\beta})|/E(2^+_{\gamma}) \leq 0.025$ [44] are ^{158}Gd , ^{178}Hf and ^{250}Cf . Comparing these results with those of [14], one can observe that our calculations with Tietz-Hua potential do not predict the additional two nuclei in [14], namely: ^{158}Dy and ^{236}U . This is due to the above cited discrepancies of the present model. However, as it was also mentioned above, one can remedy to this problem by reconsidering the present study within DDMF [12, 13, 16]. But, here we have to notice that the nucleus ^{170}Er which has been already predicted in [14] as belonging to Alhassid-Whelan arc of regularity is inconsistent with the experimental results, while our predictions for such a nucleus are coherent with these latter.

5 Conclusion

In this work, new solutions for the Bohr Hamiltonian are obtained with Tietz-Hua potential, here used as a recall potential for β -vibration and a harmonic oscillator one for γ -vibration. The

calculated energy spectra for several γ -unstable and axially symmetric prolate nuclei, within the present model, are satisfactory in comparison with the experimental data. But, as it has been mentioned above, such calculations could be improved in the framework of the deformation dependent mass formalism as it has been proved with Davidson potential in Ref [41]. As to the transition probabilities, it has been shown that Tietz-Hua potential presents an absolute prevalence when reproducing the experimental results. Such a fact is due to the flatness of this potential for large values of the β variable in comparison with other model potentials.

Table 1: The comparison of theoretical predictions of the Bohr Hamiltonian with Tietz-Hua potential to experimental data for the ground state bandhead $R_{4/2} = E(4_g^+)/E(2_g^+)$ ratios, as well as those of the β and γ bandheads, normalized to the 2_g^+ state and labelled by $R_{0/2} = E(0_\beta^+)/E(2_g^+)$ and $R_{2/2} = E(2_\gamma^+)/E(2_g^+)$ respectively. L_g , L_β and L_γ characterized the angular momenta of the highest levels of the ground state, β and γ bands respectively, included in the fit.

nucleus	$R_{4/2}$ exp	$R_{4/2}$ th	$R_{0/2}$ exp	$R_{0/2}$ th	$R_{2/2}$ exp	$R_{2/2}$ th	β_e	b_h	L_g	L_β	L_γ	n	σ
⁹⁸ Ru	2.142	2.334	2.026	3.951	2.168	2.334	8.6	0.001	24	0	4	15	0.965
¹⁰⁰ Ru	2.273	2.411	2.095	5.671	2.525	2.411	12.2	0.001	28	0	4	17	1.584
¹⁰² Ru	2.329	2.338	1.986	4.016	2.322	2.338	8.7	0.002	16	0	5	12	0.742
¹⁰⁴ Ru	2.482	2.408	2.761	3.235	2.495	2.408	4.8	0.621	8	2	8	12	0.472
¹⁰² Pd	2.293	2.419	2.863	5.995	2.758	2.419	12.2	0.01	26	4	4	18	1.954
¹⁰⁴ Pd	2.381	2.338	2.399	4016	2.414	2.338	8.7	0.002	18	2	4	13	0.750
¹⁰⁶ Pd	2.402	2.310	2.215	3.641	2.204	2.310	7.9	0.002	16	4	5	14	0.808
¹⁰⁸ Pd	2.416	2.426	2.426	2.990	2.146	2.426	4.9	0.695	14	4	4	12	0.500
¹¹⁰ Pd	2.463	2.364	2.533	1.364	2.177	2.364	0.01	0.901	12	10	4	14	0.883
¹¹² Pd	2.533	2.324	2.553	2.529	2.113	2.324	4.1	0.565	6	0	3	5	0.485
¹¹⁴ Pd	2.563	2.390	2.622	5.043	2.088	2.390	10.9	0.001	16	0	11	18	0.986
¹¹⁶ Pd	2.579	2.450	3.262	3.610	2.168	2.450	5.4	0.699	16	0	9	16	0.770
¹⁰⁶ Cd	2.361	2.384	2.838	2.848	2.713	2.384	4.5	0.621	12	0	2	7	0.225
¹⁰⁸ Cd	2.383	2.355	2.718	4.280	2.531	2.355	9.3	0.001	24	0	5	16	1.262
¹¹⁰ Cd	2.345	2.263	2.240	3.181	2.244	2.263	6.9	0.002	16	6	5	15	0.797
¹¹² Cd	2.292	2.202	1.983	2.750	2.125	2.202	5.9	0.004	12	8	11	20	0.574
¹¹⁴ Cd	2.299	2.210	2.032	2.802	2.166	2.210	6.0	0.005	14	4	3	11	0.654
¹¹⁶ Cd	2.375	2.263	2.498	3.181	2.362	2.263	6.9	0.002	14	2	3	10	0.365
¹¹⁸ Cd	2.388	2.283	2.636	3.364	2.603	2.283	7.3	0.002	14	0	3	9	0.413
¹²⁰ Cd	2.379	2.307	2.745	3.608	2.615	2.307	7.8	0.003	16	0	2	9	0.530
¹¹⁸ Xe	2.402	2.447	2.462	3.404	2.751	2.447	5.3	0.709	16	4	10	19	0.694
¹²⁰ Xe	2.468	2.442	2.817	7.261	2.716	2.442	14.5	0.01	26	4	9	23	1.764
¹²² Xe	2.501	2.464	3.470	3.423	2.546	2.464	5.7	0.763	16	0	9	16	0.831
¹²⁴ Xe	2.482	2.419	3.583	5.995	2.391	2.419	12.2	0.01	20	2	11	21	1.069
¹²⁶ Xe	2.424	2.418	3.381	3.837	2.264	2.418	5.1	0.565	12	4	9	16	0.655
¹²⁸ Xe	2.333	2.310	3.574	3.641	2.189	2.310	7.9	0.002	10	2	7	12	0.451
¹³⁰ Xe	2.247	2.296	3.346	3.489	2.093	2.296	7.6	0.001	14	0	5	11	0.479
¹³² Xe	2.157	2.070	2.771	2.081	1.944	2.070	3.5	0.227	6	0	5	7	0.353
¹³⁴ Xe	2.044	1.892	1.932	1.624	1.905	1.892	2.8	0.261	6	0	5	7	0.166
¹³⁰ Ba	2.524	2.461	3.300	3.322	2.541	2.461	5.6	0.761	12	0	6	11	0.416
¹³² Ba	2.428	2.361	3.237	4.396	2.221	2.361	9.5	0.002	14	0	8	14	0.838
¹³⁴ Ba	2.316	2.199	2.911	2.734	1.931	2.199	5.9	0.002	8	0	4	7	0.344
¹³⁶ Ba	2.280	2.013	1.929	1.886	1.895	2.013	3.2	0.264	6	0	2	4	0.181
¹⁴² Ba	2.322	2.440	4.270	4.388	3.960	2.440	5.5	0.580	14	0	2	8	0.610
¹³⁴ Ce	2.563	2.487	3.748	2.366	2.360	2.487	6.9	0.901	34	2	8	25	6.310
¹³⁶ Ce	2.380	2.276	1.949	3.295	1.978	2.276	7.1	0.004	16	0	3	10	0.715

Table 1: (continued)

nucleus	$R_{4/2}$ exp	$R_{4/2}$ th	$R_{0/2}$ exp	$R_{0/2}$ th	$R_{2/2}$ exp	$R_{2/2}$ th	β_e	b_h	L_g	L_β	L_γ	n	σ
^{138}Ce	2.316	2.132	1.873	2.396	1.915	2.132	5.1	0.003	14	0	2	8	0.401
^{140}Nd	2.329	2.126	1.827	1.823	1.926	2.126	3.3	0.490	6	0	2	4	0.165
^{148}Nd	2.493	2.434	3.039	4.491	4.139	2.434	5.5	0.541	12	8	4	13	0.996
^{140}Sm	2.347	2.380	1.867	1.863	2.676	2.380	4.2	0.749	8	0	2	5	0.152
^{142}Sm	2.332	2.237	1.888	1.947	2.158	2.237	3.6	0.577	8	0	2	5	0.160
^{142}Gd	2.346	2.321	2.656	3.781	1.902	2.321	8.2	0.002	16	0	2	9	0.503
^{144}Gd	2.348	2.349	2.540	2.530	2.525	2.349	4.2	0.607	6	0	2	4	0.102
^{152}Gd	2.194	2.417	1.788	3.604	3.222	2.417	5.0	0.595	16	10	7	19	1.001
^{154}Dy	2.233	2.389	1.975	5.022	3.070	2.389	10.8	0.002	26	10	7	24	1.942
^{156}Er	2.315	2.372	2.701	4.612	2.701	2.372	10.0	0.001	20	4	5	16	0.975
^{186}Pt	2.560	2.485	2.462	4.508	3.170	2.485	7.0	0.802	26	6	10	25	1.284
^{188}Pt	2.526	2.456	3.007	3.466	2.280	2.456	5.5	0.733	16	2	4	12	0.358
^{190}Pt	2.492	2.362	3.113	4.422	2.020	2.362	9.6	0.001	18	2	6	15	0.724
^{192}Pt	2.479	2.400	3.776	3.797	1.935	2.400	5.0	0.511	10	0	8	12	0.717
^{194}Pt	2.470	2.404	3.858	4.247	1.894	2.404	5.3	0.443	10	4	5	11	0.729
^{196}Pt	2.465	2.405	3.192	3.313	1.936	2.405	4.8	0.600	10	2	6	11	0.711
^{198}Pt	2.419	2.279	2.246	2.567	1.902	2.279	4.0	0.472	6	2	4	7	0.398
^{200}Pt	2.347	2.017	2.378	1.959	1.845	2.017	3.4	0.163	4	0	4	5	0.337

Table 2: The comparison of theoretical predictions of the axially symmetric prolate deformed nuclei of the Bohr Hamiltonian with Tietz-Hua potential to experimental data (nuclei with $R_{4/2} > 2.9$) for the ground state bandhead $R_{4/2} = E(4_{g.s.}^+)/E(2_{g.s.}^+)$ ratios, as well as those of the β and γ bandheads, normalized to the $2_{g.s.}^+$ state and labelled by $R_{0/2} = E(0_{\beta}^+)/E(2_{g.s.}^+)$ and $R_{2/2} = E(2_{\gamma}^+)/E(2_{g.s.}^+)$ respectively. L_g , L_{β} and L_{γ} characterized the angular momenta of the highest levels of the ground state, β and γ bands respectively, included in the fit.

nucleus	$R_{4/2}$ exp	$R_{4/2}$ th	$R_{0/2}$ exp	$R_{0/2}$ th	$R_{2/2}$ exp	$R_{2/2}$ th	β_e	b_h	c	L_g	L_{β}	L_{γ}	n	σ
^{150}Nd	2.93	3.14	5.19	7.36	8.16	8.60	6.8	0.003	3.5	14	6	4	13	1.083
^{152}Sm	3.01	3.18	5.62	8.38	8.92	10.52	7.8	0.001	4.3	16	14	9	23	1.716
^{154}Sm	3.25	3.30	13.41	14.01	17.57	18.59	8.2	0.351	6.7	16	6	7	17	0.601
^{154}Gd	3.01	3.24	5.53	11.15	8.10	8.45	11.4	0.001	3.0	26	26	7	32	4.230
^{156}Gd	3.24	3.28	11.79	16.15	12.97	14.29	16.3	0.001	5.1	26	12	16	34	2.639
^{158}Gd	3.29	3.30	15.05	15.48	14.93	15.16	8.4	0.330	5.3	12	6	6	14	0.242
^{160}Gd	3.30	3.31	17.62	17.86	13.13	13.08	8.6	0.432	4.4	16	4	8	17	0.172
^{162}Gd	3.29	3.31	19.93	19.91	11.98	11.90	9.5	0.294	4.0	14	0	4	10	0.096
^{156}Dy	2.93	3.19	4.90	8.97	6.50	7.18	9.1	0.001	2.6	20	10	13	27	2.062
^{158}Dy	3.21	3.27	10.01	15.20	9.57	9.86	15.5	0.002	3.4	28	6	8	24	2.790
^{160}Dy	3.27	3.30	14.75	15.07	11.13	11.46	8.0	0.370	3.9	12	4	13	20	0.293
^{162}Dy	3.29	3.31	17.33	17.42	11.01	10.80	8.4	0.422	3.6	18	8	14	26	0.324
^{164}Dy	3.30	3.31	22.56	22.82	10.38	10.21	22.9	0.003	3.4	20	0	10	19	0.199
^{166}Dy	3.31	3.32	15.00	15.07	11.19	11.15	8.0	0.554	3.7	6	2	5	8	0.044
^{160}Er	3.10	3.25	7.10	11.83	6.79	7.01	12.2	0.002	2.4	26	2	5	18	2.494
^{162}Er	3.23	3.29	10.66	11.33	8.83	9.21	6.9	0.485	3.1	20	4	12	23	0.866
^{164}Er	3.28	3.31	13.64	14.92	9.41	8.52	7.7	0.497	2.8	22	10	18	23	1.297
^{166}Er	3.29	3.30	18.12	17.99	9.75	9.80	10.5	0.154	3.3	16	10	14	26	0.176
^{168}Er	3.31	3.32	15.25	15.46	10.29	10.04	8.1	0.585	3.3	18	6	8	19	0.203
^{170}Er	3.31	3.33	11.34	9.86	11.88	11.03	8.6	0.801	3.6	24	10	19	35	2.056
^{164}Yb	3.13	3.24	7.91	11.21	7.01	7.47	11.5	0.002	2.6	18	0	5	13	1.268
^{166}Yb	3.23	3.28	10.19	14.69	9.11	9.56	15.1	0.001	3.3	24	10	13	29	2.662
^{168}Yb	3.27	3.30	13.17	20.70	11.22	11.25	21.2	0.001	3.8	34	4	7	25	4.356
^{170}Yb	3.29	3.31	12.69	13.93	13.60	13.51	7.8	0.483	4.6	20	10	17	31	0.545
^{172}Yb	3.31	3.32	13.25	15.06	18.62	18.80	8.5	0.555	6.4	16	10	5	17	0.894
^{174}Yb	3.31	3.32	19.45	19.83	21.37	21.58	9.4	0.442	7.4	20	4	5	16	0.306
^{176}Yb	3.31	3.32	13.87	13.98	15.35	15.38	8.1	0.567	5.2	20	2	5	15	0.260
^{178}Yb	3.31	3.28	15.66	15.77	14.54	14.48	15.2	0.007	5.2	6	4	2	6	0.100
^{168}Hf	3.11	3.25	7.59	11.71	7.06	7.51	12.1	0.001	2.6	22	4	4	16	2.175
^{170}Hf	3.19	3.29	8.73	17.46	9.54	9.73	18.1	0.002	3.3	34	4	4	22	5.544
^{172}Hf	3.25	3.30	9.15	20.22	11.29	11.50	20.7	0.001	3.9	38	4	6	26	5.784
^{174}Hf	3.27	3.29	9.10	17.11	13.48	13.91	17.2	0.002	4.9	26	4	5	19	3.671
^{176}Hf	3.29	3.31	13.02	14.23	15.18	15.90	8.0	0.451	5.5	18	10	8	21	0.656
^{178}Hf	3.29	3.30	12.87	13.72	12.61	12.89	7.7	0.460	4.4	18	6	6	17	0.398

Table 2: (continued)

nucleus	$R_{4/2}$ exp	$R_{4/2}$ th	$R_{0/2}$ exp	$R_{0/2}$ th	$R_{2/2}$ exp	$R_{2/2}$ th	β_e	b_h	c	L_g	L_β	L_γ	n	σ
^{180}Hf	3.31	3.31	11.81	12.19	12.86	13.05	7.6	0.580	4.4	12	4	5	12	0.226
^{176}W	3.22	3.26	7.79	13.14	9.61	10.17	13.4	0.001	3.6	22	4	57	17	2.677
^{178}W	3.24	3.29	9.40	10.96	10.47	10.51	6.9	0.467	3.6	18	10	2	15	1.049
^{180}W	3.26	3.28	14.64	14.93	10.79	11.36	15.0	0.003	4.0	24	0	7	18	0.836
^{182}W	3.29	3.31	11.35	12.25	12.20	12.24	7.6	0.594	4.1	18	4	6	16	0.836
^{184}W	3.27	3.29	9.01	9.67	8.12	8.09	6.5	0.546	2.7	10	4	6	12	0.315
^{186}W	3.23	3.30	7.21	7.95	6.02	5.89	6.2	0.656	1.9	14	4	6	14	0.411
^{178}Os	3.02	3.26	4.94	6.52	6.57	7.31	5.6	0.546	2.5	16	6	5	15	1.407
^{180}Os	3.09	3.16	5.57	7.91	6.59	7.82	7.7	0.002	3.0	10	6	7	14	1.262
^{184}Os	3.20	3.26	8.70	12.81	7.87	8.37	13.2	0.001	2.9	22	0	6	16	1.183
^{186}Os	3.17	3.28	7.74	9.77	5.60	5.80	6.2	0.471	1.9	14	10	13	24	0.877
^{188}Os	3.08	3.24	7.01	7.64	4.08	4.07	5.4	0.464	1.3	12	2	7	13	0.345
^{190}Os	2.93	3.19	4.88	5.38	2.99	3.19	4.6	0.510	1.0	10	2	6	11	0.332
^{288}Ra	3.21	3.27	11.30	14.31	13.26	13.27	14.4	0.001	4.8	22	4	3	15	1.743
^{228}Th	3.23	3.28	14.40	15.05	16.78	16.97	14.9	0.001	6.3	18	2	5	14	0.519
^{230}Th	3.27	3.32	11.93	12.23	14.69	14.52	7.8	0.601	4.9	24	4	4	17	1.719
^{232}Th	3.28	3.32	14.80	6.66	15.91	13.01	7.9	0.801	4.3	30	10	12	31	5.803
^{232}U	3.29	3.31	14.53	16.49	18.22	18.23	8.6	0.431	6.3	20	10	4	18	1.057
^{234}U	3.30	3.32	18.62	20.45	21.31	21.38	9.5	0.449	7.3	28	8	7	24	1.949
^{236}U	3.30	3.31	20.32	25.18	21.17	21.25	25.3	0.001	7.4	28	8	7	24	2.159
^{238}U	3.30	3.31	20.64	26.50	23.61	24.57	26.5	0.001	8.6	30	4	27	43	1.888
^{238}Pu	3.31	3.32	21.36	21.57	23.34	23.29	9.8	0.501	7.9	26	2	4	17	0.862
^{240}Pu	3.31	3.32	20.10	20.75	26.55	26.56	9.8	0.487	9.1	26	4	4	18	1.037
^{248}Cm	3.31	3.32	24.98	25.50	24.17	24.29	10.7	0.368	8.3	28	4	2	17	1.064
^{250}Cf	3.32	3.32	27.02	27.01	24.16	24.18	14.3	0.128	8.4	8	2	4	8	0.020

Table 3: Comparison of experimental data (Exp) [42] for several B(E2) ratios of γ -unstable nuclei to predictions by the Bohr Hamiltonian with the Tietz-Hua potential (TH), Manning-Rosen potential (MR) [27], Morse potential (M) [8], Davidson potential (D) [13] and Kratzer potential (K) [12].

nucl.		$\frac{4_g \rightarrow 2_g}{2_g \rightarrow 0_g}$	$\frac{6_g \rightarrow 4_g}{2_g \rightarrow 0_g}$	$\frac{8_g \rightarrow 6_g}{2_g \rightarrow 0_g}$	$\frac{10_g \rightarrow 8_g}{2_g \rightarrow 0_g}$	$\frac{2_\gamma \rightarrow 2_g}{2_g \rightarrow 0_g}$	$\frac{2_\gamma \rightarrow 0_g}{2_g \rightarrow 0_g} \times 10^3$	$\frac{0_\beta \rightarrow 2_g}{2_g \rightarrow 0_g}$	$\frac{2_\beta \rightarrow 0_g}{2_g \rightarrow 0_g} \times 10^3$	rms
^{98}Ru	Exp	1.44(25)				1.62(61)	36.0(152)			
	TH	1.50	1.88	2.26	2.66	1.50	0.0	0.25	0.29	0.0477
	MR	1.58	2.10	2.70	3.40	1.58	0.0	0.24	11.50	0.0498
	M	1.71	2.52	3.61	5.27	1.71	0.0	0.82	4.24	0.0942
	D	1.82	2.62	3.42	4.22	1.82	0.0	1.36	3.60	0.1420
	K	1.77	2.81	4.63	8.42	1.77	0.0	1.27	27.84	0.1198
^{100}Ru	Exp	1.45(13)				0.64(12)	41.1(52)	0.98(15)		
	TH	1.48	1.78	2.04	2.29	1.48	0.0	0.13	1.40	0.2991
	MR	1.52	1.90	2.28	2.70	1.52	0.0	0.18	14.53	0.2980
	M	1.65	2.31	3.07	4.00	1.656	0.0	0.72	8.20	0.2669
	D	1.72	2.40	3.07	3.73	1.726	0.0	1.05	10.89	0.2801
	K	1.70	2.56	3.93	6.59	1.70	0.0	1.11	43.07	0.2740
^{102}Ru	Exp	1.50(24)				0.62(7)	24.8(7)	0.80(14)		
	TH	1.50	1.88	2.24	2.64	1.50	0.0	0.25	0.34	0.2582
	MR	1.58	2.10	2.70	3.40	1.58	0.0	0.24	11.50	0.2773
	M	1.73	2.53	3.54	4.86	1.73	0.0	0.88	4.61	0.2823
	D	1.78	2.54	3.28	4.01	1.78	0.0	1.27	8.70	0.3189
	K	1.77	2.82	4.68	8.67	1.77	0.0	1.29	31.06	0.3179
^{104}Ru	Exp	1.18(28)				0.63(15)	35.0(84)	0.42(7)		
	TH	1.48	1.83	2.18	2.10	1.48	0.0	0.01	4.54	0.2479
	MR	1.58	2.08	2.66	3.37	1.58	0.0	0.23	11.72	0.2619
	M	1.62	2.10	2.49	2.80	1.62	0.0	0.71	16.34	0.2795
	D	1.63	2.18	2.71	3.21	1.63	0.0	0.79	22.41	0.2883
	K	1.60	2.20	2.95	4.00	1.60	0.0	0.68	25.59	0.2713
^{102}Pd	Exp	1.56(19)				0.46(9)	128.8(735)			
	TH	1.48	1.76	2.02	2.25	1.48	0.0	0.12	1.55	0.3408
	MR	1.50	1.88	2.24	2.62	1.50	0.0	0.17	14.71	0.3470
	M	1.68	2.35	3.07	3.86	1.68	0.0	0.80	8.20	0.4079
	D	1.76	2.49	3.19	3.87	1.76	0.0	1.22	12.34	0.4375
	K	1.63	2.31	3.25	4.77	1.63	0.0	0.87	41.64	0.3900
^{104}Pd	Exp	1.36(27)				0.61(8)	33.3(74)			
	TH	1.50	1.88	2.24	2.64	1.50	0.0	0.25	0.34	0.2977
	MR	1.58	2.10	2.70	3.40	1.58	0.0	0.24	11.50	0.3289
	M	1.67	2.39	3.30	4.60	1.67	0.0	0.73	6.18	0.3655
	D	1.74	2.45	3.15	3.85	1.74	0.0	1.11	8.13	0.3947
	K	1.70	2.52	3.74	5.83	1.70	0.0	0.99	24.16	0.3779
^{106}Pd	Exp	1.63(28)				0.98(12)	26.2(31)	0.67(18)		
	TH	1.52	1.92	2.33	2.79	1.52	0.0	0.30	0.11	0.1662
	MR	1.60	2.18	2.86	3.70	1.60	0.0	0.26	10.15	0.1861
	M	1.70	2.47	3.51	5.03	1.70	0.0	0.79	4.84	0.1830
	D	1.85	2.67	3.49	4.28	1.85	0.0	1.49	5.98	0.3033
	K	1.74	2.66	4.13	6.83	1.74	0.0	1.12	22.91	0.2220
^{108}Pd	Exp	1.47(20)	2.16(28)	2.99(48)		1.43(14)	16.6(18)	1.05(13)	1.09(29)	

Table 3: (continued)

nucl.		$\frac{4_g \rightarrow 2_g}{2_g \rightarrow 0_g}$	$\frac{6_g \rightarrow 4_g}{2_g \rightarrow 0_g}$	$\frac{8_g \rightarrow 6_g}{2_g \rightarrow 0_g}$	$\frac{10_g \rightarrow 8_g}{2_g \rightarrow 0_g}$	$\frac{2_\gamma \rightarrow 2_g}{2_g \rightarrow 0_g}$	$\frac{2_\gamma \rightarrow 0_g}{2_g \rightarrow 0_g} \times 10^3$	$\frac{0_\beta \rightarrow 2_g}{2_g \rightarrow 0_g}$	$\frac{2_\beta \rightarrow 0_g}{2_g \rightarrow 0_g} \times 10^3$	rms
	TH	1.48	1.82	2.15	2.29	1.48	0.0	0.31	73.17	0.1683
	MR	1.58	2.08	2.66	3.37	1.58	0.0	0.23	11.72	0.1300
	M	1.68	2.32	2.95	3.53	1.68	0.0	0.85	9.45	0.0594
	D	1.75	2.45	3.12	3.75	1.75	0.0	1.20	15.82	0.0782
	K	1.66	2.38	3.42	5.11	1.66	0.0	0.89	30.31	0.0836
^{110}Pd	Exp	1.71(34)				0.98(24)	14.1(22)	0.64(10)		
	TH	1.28	1.40	1.46	1.50	1.28	0.0	4.65	858.14	1.0108
	MR	1.28	1.40	1.46	1.50	1.28	0.0	4.04	860.20	0.8599
	D	1.76	2.43	3.01	3.51	1.76	0.0	1.31	26.24	0.2567
	K	1.60	2.18	2.94	4.06	1.60	0.0	0.75	42.18	0.1594
^{106}Cd	Exp	1.78(25)				0.43(12)	93.0(127)			
	TH	1.50	1.88	2.24	3.02	1.50	0.0	0.00	10.79	0.3698
	MR	1.50	1.88	2.31	2.90	1.50	0.0	3.52	648.61	0.3698
	D	1.68	2.32	2.95	3.58	1.68	0.0	0.92	10.44	0.4188
	K	1.66	2.37	3.34	4.76	1.66	0.0	0.83	16.97	0.4128
^{108}Cd	Exp	1.54(24)				0.64(20)	67.7(120)			
	TH	1.50	1.85	2.20	2.56	1.50	0.0	0.22	0.53	0.2872
	MR	1.56	2.05	2.58	3.24	1.56	0.0	0.22	12.37	0.3069
	M	1.69	2.46	3.48	5.00	1.69	0.0	0.77	4.99	0.3535
	D	1.85	2.69	3.52	4.35	1.85	0.0	1.49	4.06	0.4161
	K	1.67	2.40	3.43	5.01	1.67	0.0	0.87	19.88	0.3460
^{110}Cd	Exp	1.68(24)				1.09(19)	48.9(78)		9.85(595)	
	TH	1.54	2.00	2.50	3.08	1.54	0.0	0.39	0.01	0.1182
	MR	1.64	2.32	3.15	4.26	1.64	0.0	0.29	7.77	0.1380
	M	1.74	2.62	3.90	6.05	1.74	0.0	0.84	2.76	0.1631
	D	1.99	2.97	3.93	4.87	1.99	0.0	1.98	1.61	0.2377
	K	1.85	3.14	5.63	11.54	1.85	0.0	1.52	20.99	0.1945
^{112}Cd	Exp	2.02(22)				0.50(10)	19.9(35)	1.69(48)	11.26(210)	
	TH	1.58	2.12	2.73	3.52	1.58	0.0	0.53	0.46	0.3297
	MR	1.72	2.53	3.62	5.10	1.72	0.0	0.33	4.76	0.3710
	M	1.76	2.70	4.20	4.97	1.76	0.0	0.81	1.56	0.3122
	D	2.00	2.99	3.98	4.96	2.00	0.0	1.99	0.48	0.3056
	K	1.95	3.53	6.92	15.92	1.95	0.0	1.82	12.87	0.2913
^{114}Cd	Exp	1.99(25)	3.83(72)	2.73(97)		0.71(24)	15.4(29)	0.88(11)	10.61(193)	
	TH	1.58	2.10	2.70	3.46	1.58	0.0	0.51	0.36	0.2886
	MR	1.70	2.50	3.55	4.98	1.70	0.0	0.32	5.16	0.2792
	M	1.78	2.79	4.42	3.44	1.78	0.0	0.85	0.99	0.3228
	D	2.00	2.99	3.97	4.94	2.00	0.0	1.99	0.74	0.3232
	K	1.93	3.46	6.72	15.44	1.93	0.0	1.77	15.44	0.6105
^{116}Cd	Exp	1.70(52)				0.63(46)	32.8(86)	0.02		
	TH	1.54	2.00	2.50	3.08	1.54	0.0	0.39	0.01	0.2481
	MR	1.64	2.32	3.15	4.26	1.64	0.0	0.29	7.75	0.2609
	M	1.66	2.39	3.43	5.00	1.66	0.0	0.63	4.55	0.2986
	D	1.74	2.46	3.17	3.90	1.74	0.0	1.11	4.42	0.3883
	K	1.69	2.47	3.52	5.05	1.69	0.0	0.90	10.02	0.3437
^{118}Cd	Exp	1.85						0.16		
	TH	1.53	1.96	2.42	2.94	1.53	0.0	0.35	0.01	0.1861

Table 3: (continued)

nucl.		$\frac{4_g \rightarrow 2_g}{2_g \rightarrow 0_g}$	$\frac{6_g \rightarrow 4_g}{2_g \rightarrow 0_g}$	$\frac{8_g \rightarrow 6_g}{2_g \rightarrow 0_g}$	$\frac{10_g \rightarrow 8_g}{2_g \rightarrow 0_g}$	$\frac{2_\gamma \rightarrow 2_g}{2_g \rightarrow 0_g}$	$\frac{2_\gamma \rightarrow 0_g}{2_g \rightarrow 0_g} \times 10^3$	$\frac{0_\beta \rightarrow 2_g}{2_g \rightarrow 0_g}$	$\frac{2_\beta \rightarrow 0_g}{2_g \rightarrow 0_g} \times 10^3$	rms
	MR	1.63	2.26	3.04	4.04	1.63	0.0	0.28	8.60	0.1253
	M	1.66	2.38	3.42	4.98	1.66	0.0	0.62	4.60	0.2488
	D	1.71	2.39	3.06	3.74	1.71	0.0	1.00	5.88	0.4258
	K	1.70	2.51	3.65	5.41	1.70	0.0	0.95	13.14	0.4021
^{118}Xe	Exp	1.11(7)	0.88(27)	0.49(20)	0.73					
	TH	1.46	1.76	2.06	2.35	1.46	0.0	0.33	74.23	0.6109
	MR	1.46	1.76	2.02	2.27	1.46	0.0	3.29	628.38	0.5913
	M	1.65	2.19	2.65	3.03	1.65	0.0	0.78	13.52	0.8639
	D	1.67	2.28	2.85	3.39	1.67	0.0	0.95	21.93	0.9648
	K	1.61	2.21	3.00	4.17	1.61	0.0	0.74	34.42	1.1216
^{120}Xe	Exp	1.16(14)	1.17(24)	0.96(22)	0.91(19)					
	TH	1.46	1.73	1.96	2.16	1.46	0.0	0.10	1.96	0.4296
	MR	1.44	1.72	1.91	2.08	1.44	0.0	3.24	635.04	0.4062
	M	1.58	2.05	2.49	2.90	1.58	0.0	0.54	15.06	0.6723
	D	1.60	2.11	2.60	3.08	1.60	0.0	0.67	21.51	0.7269
	K	1.56	2.06	2.64	3.43	1.56	0.0	0.62	42.25	0.7946
^{122}Xe	Exp	1.47(38)	0.89(26)	0.44						
	TH	1.46	1.75	1.98	2.24	1.46	0.0	0.79	158.83	0.5875
	MR	1.44	1.73	1.94	2.14	1.44	0.0	3.32	646.12	0.5727
	D	1.58	2.05	2.48	2.89	1.58	0.0	0.63	29.29	0.7826
	K	1.54	2.00	2.52	3.17	1.54	0.0	0.54	36.27	0.7858
^{124}Xe	Exp	1.34(24)	1.59(17)	0.63(29)	0.29(8)	0.70(19)	15.9(46)			
	TH	1.48	1.76	2.02	2.25	1.48	0.0	0.12	1.55	0.4212
	MR	1.46	1.73	1.96	2.18	1.46	0.0	3.29	637.34	0.4052
	M	1.57	2.05	2.50	2.92	1.57	0.0	0.54	14.87	0.5622
	D	1.59	2.09	2.57	3.04	1.59	0.0	0.63	20.14	0.5862
	K	1.55	2.03	2.57	3.25	1.55	0.0	0.53	28.40	0.6106
^{128}Xe	Exp	1.47(20)	1.94(26)	2.39(40)	2.74(114)	1.19(19)	15.9(23)			
	TH	1.52	1.92	2.33	2.79	1.52	0.0	0.30	0.11	0.0571
	MR	1.60	2.16	2.86	3.76	1.60	0.0	0.25	10.10	0.2031
	M	1.60	2.19	2.90	3.89	1.60	0.0	0.55	8.95	0.2249
	D	1.63	2.20	2.75	3.31	1.63	0.0	0.73	9.64	0.1428
	K	1.83	2.95	4.73	7.64	1.83	0.0	0.75	12.57	0.9281
^{132}Xe	Exp	1.24(18)				1.77(29)	3.4(7)			
	TH	1.66	2.40	3.28	8.85	1.66	0.0	0.88	3.61	0.1444
	MR	1.78	2.80	0.36	0.34	1.78	0.0	0.22	1.07	0.1794
	D	2.00	3.00	4.00	5.00	2.00	0.0	2.00	0.00	0.2638
	K	2.78	7.13	17.89	43.35	2.78	0.0	2.49	0.07	0.6129
^{130}Ba	Exp	1.36(6)	1.62(15)	1.55(56)	0.93(15)					
	TH	1.46	1.75	2.00	2.27	1.46	0.0	0.79	159.53	0.3548
	MR	1.46	1.73	1.94	2.14	1.46	0.0	3.33	647.59	0.3191
	M	1.66	2.28	2.90	3.50	1.66	0.0	0.78	10.07	0.7469
	D	1.56	2.01	2.41	2.77	1.56	0.0	0.61	34.54	0.5183
	K	1.54	2.01	2.54	3.22	1.54	0.0	0.56	39.43	0.6318
^{132}Ba	Exp					3.35(64)	90.7(177)			
	TH	1.48	1.85	2.18	2.52	1.48	0.0	0.21	0.61	0.9393
	MR	1.56	2.05	2.56	3.20	1.56	0.0	0.22	12.50	0.8993

Table 3: (continued)

nucl.		$\frac{4_g \rightarrow 2_g}{2_g \rightarrow 0_g}$	$\frac{6_g \rightarrow 4_g}{2_g \rightarrow 0_g}$	$\frac{8_g \rightarrow 6_g}{2_g \rightarrow 0_g}$	$\frac{10_g \rightarrow 8_g}{2_g \rightarrow 0_g}$	$\frac{2_\gamma \rightarrow 2_g}{2_g \rightarrow 0_g}$	$\frac{2_\gamma \rightarrow 0_g}{2_g \rightarrow 0_g} \times 10^3$	$\frac{0_\beta \rightarrow 2_g}{2_g \rightarrow 0_g}$	$\frac{2_\beta \rightarrow 0_g}{2_g \rightarrow 0_g} \times 10^3$	rms
	M	1.48	1.84	2.19	2.66	1.48	0.0	0.00	0.00	0.9393
	D	1.68	2.30	2.90	3.50	1.68	0.0	0.92	15.21	0.8394
	K	1.61	2.20	2.94	3.95	1.61	0.0	0.66	20.59	0.8744
^{134}Ba	Exp	1.55(21)				2.17(69)	12.5(41)			
	TH	1.58	2.12	2.75	3.54	1.58	0.0	0.53	0.50	0.1992
	MR	1.43	1.68	1.86	1.98	1.43	0.0	0.05	8.86	0.2523
	M	1.70	2.53	3.81	1.96	1.70	0.0	0.68	2.48	0.1665
	D	1.75	2.48	3.21	3.94	1.75	0.0	1.14	4.08	0.1569
	K	2.13	4.10	7.88	15.19	2.13	0.0	1.26	6.22	0.1933
^{142}Ba	Exp	1.40(17)	0.56(14)							
	TH	1.46	1.76	2.04	2.35	1.46	0.0	0.05	0.39	0.6000
	MR	1.46	1.76	2.02	2.29	1.46	0.0	3.33	635.55	0.6000
	M	1.50	1.89	2.30	2.88	1.50	0.0	0.11	2.37	0.6661
	D	1.55	2.00	2.41	2.82	1.55	0.0	0.49	18.60	0.7231
	K	1.54	1.99	2.46	3.04	1.54	0.0	0.45	21.34	0.7176
^{148}Nd	Exp	1.61(13)	1.76(19)			0.25(4)	9.3(17)	0.54(6)	32.82(816)	
	TH	1.48	1.78	2.06	2.35	1.48	0.0	0.07	0.67	0.2204
	MR	1.56	2.05	2.58	3.22	1.56	0.0	0.22	12.41	0.2296
	M	1.63	2.14	2.56	2.89	1.63	0.0	0.73	15.09	0.2398
	D	1.63	2.17	2.68	3.15	1.63	0.0	0.81	26.86	0.2432
	K	1.57	2.08	2.67	3.47	1.57	0.0	0.59	30.88	0.2259
^{152}Gd	Exp	1.84(29)	2.74(81)			0.23(4)	4.2(8)	2.47(78)		
	TH	1.48	1.82	2.15	2.42	1.48	0.0	0.04	0.02	0.5827
	MR	1.62	2.20	2.91	3.81	1.62	0.0	0.26	9.67	0.5363
	M	1.71	2.47	3.39	4.54	1.71	0.0	0.85	5.63	0.4438
	D	1.98	2.92	3.81	4.65	1.98	0.0	1.95	4.51	0.3674
	K	1.80	2.96	5.14	10.30	1.80	0.0	1.41	32.70	0.3815
^{154}Dy	Exp	1.62(35)	2.05(42)	2.27(62)	1.86(69)					
	TH	1.48	1.82	2.10	2.38	1.48	0.0	0.17	1.03	0.1520
	MR	1.53	1.95	2.38	2.87	1.53	0.0	0.19	13.92	0.2545
	M	1.67	2.37	3.20	4.25	1.67	0.0	0.76	7.08	0.6439
	D	1.91	2.79	3.64	4.46	1.91	0.0	1.70	5.41	0.7586
	K	1.78	2.89	5.06	10.73	1.78	0.0	1.46	58.09	2.3322
^{156}Er	Exp	1.78(16)	1.89(36)	0.76(20)	0.88(22)					
	TH	1.48	1.83	2.15	2.46	1.48	0.0	0.19	0.76	0.5310
	MR	1.54	2.00	2.50	3.06	1.54	0.0	0.21	13.12	0.6997
	M	1.62	2.23	2.91	3.74	1.62	0.0	0.64	9.60	0.8986
	D	1.70	2.35	3.00	3.64	1.70	0.0	0.98	11.50	0.8954
	K	1.64	2.33	3.27	4.73	1.64	0.0	0.83	28.76	1.1540
^{192}Pt	Exp	1.56(12)	1.23(55)			1.91(16)	9.5(9)			
	TH	1.48	1.83	2.16	2.48	1.48	0.0	0.11	0.44	0.1848
	MR	1.48	1.83	2.18	2.62	1.48	0.0	3.40	632.98	0.1848
	M	1.58	2.08	2.55	3.00	1.58	0.0	0.56	14.25	0.2269
	D	1.59	2.09	2.57	3.05	1.59	0.0	0.61	16.98	0.2284
	K	1.57	2.09	2.68	3.44	1.57	0.0	0.54	17.79	0.2301
^{194}Pt	Exp	1.73(13)	1.36(45)	1.02(30)	0.69	1.81(25)	5.9(9)	0.01		
	TH	1.48	1.82	2.13	2.48	1.48	0.0	0.12	0.75	0.3129

Table 3: (continued)

nucl.		$\frac{4_g \rightarrow 2_g}{2_g \rightarrow 0_g}$	$\frac{6_g \rightarrow 4_g}{2_g \rightarrow 0_g}$	$\frac{8_g \rightarrow 6_g}{2_g \rightarrow 0_g}$	$\frac{10_g \rightarrow 8_g}{2_g \rightarrow 0_g}$	$\frac{2_\gamma \rightarrow 2_g}{2_g \rightarrow 0_g}$	$\frac{2_\gamma \rightarrow 0_g}{2_g \rightarrow 0_g} \times 10^3$	$\frac{0_\beta \rightarrow 2_g}{2_g \rightarrow 0_g}$	$\frac{2_\beta \rightarrow 0_g}{2_g \rightarrow 0_g} \times 10^3$	rms
	MR	1.58	2.06	2.64	3.35	1.58	0.0	0.22	11.82	0.4564
	M	1.58	2.06	2.49	2.88	1.58	0.0	0.58	15.34	0.3829
	D	1.59	2.09	2.57	3.04	1.59	0.0	0.63	19.78	0.4073
	K	1.56	2.07	2.63	3.34	1.56	0.0	0.52	19.45	0.4503
^{196}Pt	Exp	1.48(3)	1.80(23)	1.92(23)			0.4	0.07(4)	0.06(6)	
	TH	1.48	1.83	2.18	2.14	1.48	0.0	0.04	0.41	0.0436
	MR	1.48	1.83	2.18	2.62	1.48	0.0	3.38	629.69	0.5626
	M	1.63	2.14	2.57	2.93	1.63	0.0	0.72	14.74	0.1644
	D	1.64	2.21	2.75	3.28	1.64	0.0	0.82	20.83	0.1995
	K	1.61	2.21	2.97	4.04	1.61	0.0	0.69	23.11	0.2147
^{198}Pt	Exp	1.19(13)	1.78			1.16(23)	1.2(4)	0.81(22)	1.56(126)	
	TH	1.54	2.00	1.84	3.16	1.54	0.0	0.27	0.02	0.1298
	MR	1.74	2.64	4.00	6.20	1.74	0.0	0.33	3.27	0.2113
	M	1.72	2.48	3.35	4.36	1.72	0.0	0.89	5.78	0.1737
	D	1.82	2.60	3.36	4.08	1.82	0.0	1.41	10.09	0.2272
	K	1.76	2.73	4.24	6.76	1.76	0.0	1.16	11.09	0.2176

Table 4: Comparison of experimental data [42] (upper line) for several B(E2) ratios of axially symmetric prolate deformed nuclei to predictions (lower line) by the Bohr Hamiltonian with the Tietz-Hua potential (TH), Manning-Rosen potential (MR) [27], Morse potential (M) [8], Davidson potential (D) [13] and Kratzer potential (K) [12]

nucl.		$\frac{4_g \rightarrow 2_g}{2_g \rightarrow 0_g}$	$\frac{6_g \rightarrow 4_g}{2_g \rightarrow 0_g}$	$\frac{8_g \rightarrow 6_g}{2_g \rightarrow 0_g}$	$\frac{10_g \rightarrow 8_g}{2_g \rightarrow 0_g}$	$\frac{2_{\beta} \rightarrow 0_g}{2_g \rightarrow 0_g} \times 10^3$	$\frac{2_{\beta} \rightarrow 2_g}{2_g \rightarrow 0_g} \times 10^3$	$\frac{2_{\beta} \rightarrow 4_g}{2_g \rightarrow 0_g} \times 10^3$	$\frac{2_{\gamma} \rightarrow 0_g}{2_g \rightarrow 0_g} \times 10^3$	$\frac{2_{\gamma} \rightarrow 2_g}{2_1 \rightarrow 0_g} \times 10^3$	$\frac{2_{\gamma} \rightarrow 4_g}{2_g \rightarrow 0_g} \times 10^3$	rms
¹⁵⁰ Nd	Exp	1.52(4)	1.84(14)	2.05(13)		4.4(8)	61.7(98)	174(55)	26.1(22)	49.6(26)	14.8(98)	
	TH	1.49	1.77	2.05	2.42	7.06	28.56	199.77	81.16	125.16	7.27	0,0144
	MR	1.53	1.91	2.36	2.97	25.36	69.51	349.03	6.25	136.16	8.07	0.0414
	K	1.55	1.98	2.55	3.40	36.4	93.5	443	95.9	150.0	9.0	0.0664
¹⁵² Sm	Exp	1.45(5)	1.70(7)	1.98(14)	2.22(25)	6.4(7)	38.2(43)	132(15)	25.1(17)	64.6(48)	5.4(5)	
	TH	1.51	1.84	2.22	2.70	25.17	62.89	284.15	69.86	107.33	6.22	0,0085
	MR	1.51	1.84	2.22	2.70	25.17	62.89	284.15	69.86	107.33	6.22	0.0566
	K	1.55	1.98	2.58	3.53	49.1	113.4	475	84.0	130.9	7.8	0.1501
¹⁵⁴ Sm	Exp	1.40(5)	1.67(7)	1.83(11)	1.81(11)	5.4(13)		25(6)	18.4(34)		3.9(7)	
	TH	1.44	1.61	1.73	1.84	3.53	7.98	32.42	45.92	66.63	3.45	0,0166
	MR	1.46	1.68	1.86	2.05	20.83	40.11	127.16	43.52	63.90	3.39	0.0336
	M	1.46	1.67	1.85	2.03	21.1	39.8	122	47.3	69.2	3.6	0.0311
	D	1.47	1.69	1.87	2.06	26.7	50.0	150	47.5	69.6	3.7	0.0364
	K	1.46	1.67	1.86	2.05	24.7	45.7	136	47.8	69.9	3.7	0.0341
¹⁵⁴ Gd	Exp	1.56(7)	1.82(11)	1.99(12)	2.29(27)	5.5(5)	42.7(41)	125(11)	36.3(34)	78.3(69)	11.0(10)	
	TH	1.46	1.66	1.82	1.99	5.44	16.06	85.48	108.27	158.51	8.32	0,0415
	MR	1.48	1.74	1.99	2.28	23.43	50.01	184.53	116.96	172.02	9.12	0.0183
	K	1.55	1.98	2.57	3.54	55.4	122.5	486	114.7	175.6	10.1	0.1439
¹⁵⁶ Gd	Exp	1.41(5)	1.58(6)	1.71(10)	1.68(9)	3.4(3)	18(2)	22(2)	25.0(15)	38.7(24)	4.1(3)	
	TH	1.44	1.62	1.74	1.85	4.77	11.16	46.02	63.13	91.48	4.72	0,0183
	MR	1.46	1.66	1.82	1.99	19.71	36.77	110.87	67.06	97.41	5.05	0.0352
	M	1.47	1.70	1.90	2.12	22.5	44.3	145	63.0	92.4	4.9	0.0508
	D	1.48	1.73	1.95	2.18	29.7	59.1	191	62.5	92.4	4.9	0.0599
	K	1.47	1.69	1.90	2.13	30.8	56.5	166	70.7	103.3	5.4	0.0523
¹⁵⁸ Gd	Exp	1.46(5)		1.67(16)	1.72(16)	1.6(2)	0.4(1)	7.0(8)	17.2(20)	30.3(45)	1.4(2)	
	TH	1.44	1.61	1.72	1.83	3.66	8.12	31.95	61.06	88.20	4.52	0,0160
	MR	1.46	1.67	1.84	2.03	20.39	38.75	120.37	58.94	85.98	4.50	0.0420
	M	1.45	1.64	1.79	1.95	14.6	26.7	79	63.3	91.7	4.7	0.0311
	D	1.46	1.66	1.82	1.98	25.7	45.9	127	64.0	93.0	4.8	0.0372
	K	1.45	1.66	1.82	1.98	24.1	42.8	119	63.9	92.6	4.8	0.0368
¹⁵⁶ Dy	Exp	1.75(14)	1.34(12)	1.94(13)	2.45(21)				48.2(35)	63.0(78)	84.4(141)	
	TH	1.47	1.70	1.91	2.15	5.77	20.40	127.80	123.46	183.16	9.84	0,0815
	MR	1.50	1.81	2.14	2.56	24.86	59.14	251.74	135.31	201.86	10.99	0.0865
	K	1.56	2.03	2.70	3.83	53.2	124.3	531	151.8	232.6	13.4	0.2487
¹⁵⁸ Dy	Exp	1.45(10)	1.86(12)	1.86(38)	1.75(28)	12(3)	19(4)	66(16)	32.2(78)	103.8(258)	11.5(48)	
	TH	1.44	1.62	1.75	1.86	4.73	11.51	49.79	97.07	140.40	7.20	0,0298
	MR	1.46	1.67	1.85	2.03	20.40	38.78	120.48	99.94	145.02	7.49	0.0351
	M	1.48	1.73	1.98	2.26	23.5	49.0	175	95.1	140.1	7.5	0.0554
	D	1.50	1.78	2.04	2.31	30.5	65.4	232	88.5	131.7	7.1	0.0620
	K	1.48	1.73	1.98	2.28	32.5	63.0	202	97.9	143.6	7.6	0.0580
¹⁶⁰ Dy	Exp	1.46(7)	1.23(7)	1.70(16)	1.69(9)	3.4(4)		8.5(10)	23.2(21)	43.8(42)	3.1(3)	
	TH	1.44	1.61	1.72	1.83	3.44	7.67	30.48	84.81	122.21	6.23	0,0455
	MR	1.45	1.65	1.79	1.94	18.59	33.73	97.21	84.80	122.55	6.28	0.0561
	M	1.46	1.67	1.84	2.02	20.9	39.3	120	82.7	120.1	6.2	0.0640
	D	1.46	1.68	1.85	2.03	22.9	43.5	133	78.6	114.5	6.0	0.0659
	K	1.46	1.66	1.83	2.00	23.5	42.5	122	87.4	126.6	6.5	0.0619
¹⁶² Dy	Exp	1.45(7)	1.51(10)	1.74(10)	1.76(13)				0.12(1)	0.20	0.02	
	TH	1.44	1.60	1.70	1.79	2.85	5.87	20.86	92.67	133.05	6.73	0,0275
	MR	1.45	1.65	1.79	1.94	18.59	33.73	97.21	84.80	122.55	6.28	0.0394
	M	1.45	1.62	1.76	1.88	11.4	20.1	56	94.3	135.6	6.9	0.0331

Table 4: (continued)

nucl.		$\frac{4g \rightarrow 2g}{2g \rightarrow 0g}$	$\frac{6g \rightarrow 4g}{2g \rightarrow 0g}$	$\frac{8g \rightarrow 6g}{2g \rightarrow 0g}$	$\frac{10g \rightarrow 8g}{2g \rightarrow 0g}$	$\frac{2\beta \rightarrow 0g}{2g \rightarrow 0g} \times 10^3$	$\frac{2\beta \rightarrow 2g}{2g \rightarrow 0g} \times 10^3$	$\frac{2\beta \rightarrow 4g}{2g \rightarrow 0g} \times 10^3$	$\frac{2\gamma \rightarrow 0g}{2g \rightarrow 0g} \times 10^3$	$\frac{2\gamma \rightarrow 2g}{2g \rightarrow 0g} \times 10^3$	$\frac{2\gamma \rightarrow 4g}{2g \rightarrow 0g} \times 10^3$	rms
	D	1.45	1.65	1.80	1.95	23.9	42.4	116	89.8	129.8	6.7	0.0412
	K	1.45	1.65	1.80	1.95	23.7	41.4	112	92.3	133.2	6.8	0.0416
^{164}Dy	Exp	1.30(7)	1.56(7)	1.48(9)	1.69(9)				19.1(22)	38.3(39)	4.6(5)	
	TH	1.44	1.60	1.69	1.77	3.70	7.54	25.90	98.19	140.90	7.11	0.0407
	MR	1.44	1.61	1.74	1.85	15.91	27.35	71.78	99.64	143.21	7.26	0.0501
	M	1.44	1.62	1.74	1.85	16.1	27.6	72	99.7	143.2	7.3	0.0503
	D	1.44	1.62	1.75	1.86	16.9	29.1	77	99.7	143.4	7.3	0.0519
	K	1.45	1.64	1.79	1.93	23.6	40.6	107	100.4	144.7	7.4	0.0619
^{162}Er	Exp					8(7)		170(90)	32.5(28)	77.0(56)	9.4(69)	
	TH	1.44	1.62	1.75	1.88	2.21	5.32	23.87	107.06	154.56	7.90	0.0365
	MR	1.47	1.70	1.90	2.13	21.93	43.82	146.95	96.69	141.23	7.40	0.0190
	M	1.47	1.72	1.95	2.21	22.0	45.1	158	96.6	141.7	7.5	0.0186
	D	1.49	1.75	1.99	2.24	27.8	58.3	202	91.1	134.8	7.2	0.0180
	K	1.48	1.73	1.97	2.25	28.9	57.1	189	100.4	147.1	7.8	0.0202
^{164}Er	Exp	1.18(13)		1.57(9)	1.64(11)				23.9(35)	52.3(72)	7.8(12)	
	TH	1.44	1.60	1.71	1.80	2.21	4.69	17.50	119.45	171.50	8.67	0.0607
	MR	1.46	1.66	1.81	1.98	19.36	35.81	106.42	99.97	144.61	7.42	0.0850
	M	1.47	1.69	1.87	2.05	23.4	44.8	139	103.3	150.3	7.8	0.0988
	D	1.47	1.70	1.89	2.09	28.3	53.5	162	103.8	151.2	7.9	0.1051
	K	1.46	1.67	1.86	2.05	27.0	49.0	141	110.5	160.2	8.2	0.0976
^{166}Er	Exp	1.45(12)	1.62(22)	1.71(25)	1.73(23)				25.7(31)	45.3(54)	3.1(4)	
	TH	1.44	1.61	1.72	1.81	4.08	9.01	34.81	100.86	145.15	7.37	0.0212
	MR	1.45	1.65	1.80	1.95	18.77	34.22	99.34	99.95	144.37	7.39	0.0381
	M	1.45	1.64	1.78	1.92	17.3	30.9	88	104.5	150.7	7.7	0.0342
	D	1.46	1.66	1.81	1.96	20.7	38.2	111	100.0	144.8	7.4	0.0400
^{168}Er	Exp	1.54(7)	2.13(16)	1.69(11)	1.46(11)				23.2(15)	41.1(31)	3.0(3)	
	TH	1.44	1.59	1.69	1.77	0.53	1.04	3.52	101.37	145.31	7.32	0.0923
	MR	1.45	1.65	1.80	1.94	18.63	33.83	97.64	94.35	136.28	6.98	0.1007
	M	1.44	1.61	1.73	1.84	1.8	3.1	8	101.1	145.1	7.3	0.0953
	D	1.45	1.65	1.79	1.93	27.7	47.2	120	100.6	145.1	7.4	0.0998
	K	1.45	1.64	1.78	1.92	27.6	46.2	116	100.6	144.9	7.4	0.0996
^{170}Er	Exp			1.78(15)	1.54(11)	1.4(1)	0.2(2)	6.8(12)	17.7(9)		1.4(4)	
	TH	1.43	1.59	1.67	1.74	276.71	404.38	767.24	92.97	133.05	6.68	0.1336
	MR	1.41	1.53	1.57	1.58	645.85	913.53	1608.76	91.50	130.70	6.53	0.2809
	M	1.46	1.68	1.84	1.98	28.1	50.6	139	76.1	110.6	5.7	0.0670
	D	1.47	1.69	1.86	2.03	39.2	67.9	177	78.6	114.2	5.9	0.0761
	K	1.46	1.66	1.83	2.01	42.8	70.7	173	84.6	122.2	6.3	0.0730
^{166}Yb	Exp	1.43(9)	1.53(10)	1.70(18)	1.61(80)							
	TH	1.45	1.63	1.75	1.87	4.81	11.92	52.72	99.96	144.71	7.43	0.0688
	MR	1.46	1.68	1.86	2.05	20.75	39.85	125.85	103.00	149.60	7.74	0.1210
	M	1.48	1.73	1.97	2.23	24.3	50.3	176	101.4	149.0	7.9	0.1746
	D	1.50	1.78	2.05	2.33	33.7	71.0	245	97.2	144.5	7.8	0.2083
	K	1.48	1.73	1.97	2.27	35.4	66.7	206	115.2	168.2	8.8	0.1834
^{168}Yb	Exp					8.6(9)			22.0(55)	45.9(73)	8.6	
	TH	1.44	1.60	1.69	1.77	3.73	7.62	26.22	87.71	125.92	6.36	0.0259
	MR	1.45	1.63	1.76	1.88	16.94	29.69	80.63	89.27	128.55	6.54	0.0267
	M	1.47	1.70	1.90	2.11	23.3	45.7	148	84.6	123.7	6.5	0.0252
	D	1.48	1.72	1.93	2.14	29.6	57.5	180	82.9	121.6	6.4	0.0248
	K	1.46	1.68	1.86	2.06	26.3	48.2	142	87.6	127.2	6.6	0.0265
^{170}Yb	Exp			1.79(16)	1.77(14)	5.4(10)			13.4(34)	23.9(57)	2.4(6)	
	TH	1.44	1.61	1.71	1.81	2.06	4.43	16.94	71.69	103.23	5.25	0.0223
	MR	1.46	1.66	1.83	1.99	19.77	36.95	111.68	61.71	89.74	4.66	0.0395
	M	1.44	1.64	1.78	1.93	8.3	15.1	44	73.6	106.3	5.4	0.0315
	D	1.47	1.71	1.91	2.12	30.6	58.2	176	66.2	97.1	5.1	0.0633
	K	1.47	1.69	1.90	2.13	31.9	58.0	168	69.4	101.3	5.3	0.0646

Table 4: (continued)

nucl.		$\frac{4g \rightarrow 2g}{2g \rightarrow 0g}$	$\frac{6g \rightarrow 4g}{2g \rightarrow 0g}$	$\frac{8g \rightarrow 6g}{2g \rightarrow 0g}$	$\frac{10g \rightarrow 8g}{2g \rightarrow 0g}$	$\frac{2\beta \rightarrow 0g}{2g \rightarrow 0g} \times 10^3$	$\frac{2\beta \rightarrow 2g}{2g \rightarrow 0g} \times 10^3$	$\frac{2\beta \rightarrow 4g}{2g \rightarrow 0g} \times 10^3$	$\frac{2\gamma \rightarrow 0g}{2g \rightarrow 0g} \times 10^3$	$\frac{2\gamma \rightarrow 2g}{2_1 \rightarrow 0g} \times 10^3$	$\frac{2\gamma \rightarrow 4g}{2g \rightarrow 0g} \times 10^3$	rms
^{172}Yb	Exp	1.42(10)	1.51(14)	1.89(19)	1.77(11)	1.1(1)	3.7(6)	12(1)	6.3(6)		0.6(1)	
	TH	1.44	1.59	1.69	1.77	0.31	0.61	2.06	51.40	73.84	3.74	0.0247
	MR	1.46	1.67	1.85	2.03	20.40	38.78	120.49	44.00	64.44	3.40	0.0369
	M	1.46	1.67	1.83	1.99	23.6	43.1	124	48.7	70.9	3.7	0.0342
	D	1.46	1.67	1.83	1.99	32.2	55.9	147	51.6	75.0	3.9	0.0355
	K	1.46	1.66	1.83	2.01	29.6	51.6	139	51.0	74.2	3.8	0.0361
^{174}Yb	Exp	1.39(7)	1.84(26)	1.93(12)	1.67(12)					12.4(29)		
	TH	1.43	1.59	1.68	1.76	2.26	4.33	13.77	44.26	63.56	3.22	0.0744
	MR	1.45	1.63	1.77	1.90	17.43	30.84	85.17	42.72	61.86	3.19	0.0718
	M	1.45	1.63	1.75	1.86	18.9	32.3	88	44.6	64.3	3.3	0.0690
	D	1.45	1.63	1.75	1.86	20.9	35.1	88	45.0	64.9	3.3	0.0691
	K	1.44	1.62	1.74	1.86	20.6	34.3	85	45.7	65.8	3.4	0.0711
^{176}Yb	Exp	1.49(15)	1.63(14)	1.65(28)	1.76(18)				9.8			
	TH	1.44	1.60	1.70	1.78	0.14	0.28	0.98	63.67	91.47	4.63	0.0191
	MR	1.45	1.64	1.78	1.91	17.89	31.96	89.72	62.34	90.11	4.63	0.0413
	M	1.44	1.62	1.74	1.87	0.5	1.0	3	65.1	93.7	4.8	0.0317
	D	1.46	1.66	1.82	1.97	27.9	49.0	132	63.1	91.6	4.7	0.0551
	K	1.45	1.65	1.81	1.97	28.6	49.0	128	64.5	93.4	4.8	0.0541
^{174}Hf	Exp					14(4)		9(3)	31.6(161)	48.7(124)		
	TH	1.44	1.61	1.73	1.83	4.57	10.43	41.63	66.39	95.98	4.92	0.0169
	MR	1.45	1.65	1.81	1.96	19.10	35.09	103.15	68.71	99.59	5.14	0.0283
	M	1.46	1.66	1.81	1.97	20.3	37.3	109	65.4	94.9	4.9	0.0288
	D	1.48	1.74	1.96	2.20	31.4	62.2	200	66.9	98.8	5.3	0.0503
	K	1.49	1.76	2.05	2.42	47.1	87.5	264	69.7	102.9	5.5	0.0663
^{176}Hf	Exp					5.4(11)		31(6)	21.3(26)			
	TH	1.44	1.61	1.71	1.81	2.41	5.19	19.80	59.11	85.25	4.35	0.0132
	MR	1.46	1.67	1.85	2.04	20.55	39.25	122.82	54.41	79.50	4.17	0.0327
	M	1.46	1.68	1.86	2.04	23.3	43.9	133	57.0	83.2	4.4	0.0363
	D	1.47	1.70	1.89	2.09	30.8	57.3	169	57.9	84.9	4.5	0.0481
	K	1.46	1.68	1.86	2.06	29.1	52.2	148	60.3	87.8	4.6	0.0417
^{178}Hf	Exp		1.38(9)	1.49(6)	1.62(7)	0.4(2)		2.4(9)	24.5(39)	27.7(28)	1.6(2)	
	TH	1.44	1.61	1.72	1.82	2.45	5.39	21.21	74.86	107.90	5.50	0.0477
	MR	1.46	1.67	1.85	2.03	20.41	38.80	120.62	72.99	106.23	5.53	0.0780
	M	1.46	1.68	1.86	2.04	23.3	44.3	136	73.5	107.2	5.6	0.0805
	D	1.47	1.69	1.88	2.07	28.4	53.1	158	73.8	107.8	5.6	0.0855
	K	1.46	1.68	1.86	2.06	27.1	49.2	142	75.7	110.0	5.7	0.0824
^{180}Hf	Exp	1.48(20)	1.41(15)	1.61(26)	1.55(10)				24.5(47)	32.9(56)		
	TH	1.44	1.60	1.71	1.80	1.12	2.05	6.18	75.36	108.38	5.50	0.0570
	MR	1.47	1.70	1.92	2.16	22.20	44.81	152.59	66.88	98.36	5.23	0.1241
	D	1.46	1.66	1.82	1.98	34.9	59.5	151	78.4	113.4	5.8	0.0911
	K	1.46	1.66	1.83	2.00	34.6	58.6	150	80.3	116.0	6.0	0.0944
^{182}W	Exp	1.43(8)	1.46(16)	1.53(14)	1.48(14)	6.6(6)	4.6(6)	13(1)	24.8(12)	24.8(12)	0.2	
	TH	1.44	1.60	1.71	1.80	2.69	4.71	13.01	81.14	116.61	5.91	0.0405
	MR	1.46	1.66	1.83	2.00	19.78	36.97	111.79	76.71	111.32	5.76	0.0647
	M	1.46	1.68	1.85	2.00	25.7	47.4	137	79.3	115.4	6.0	0.0669
	D	1.47	1.69	1.87	2.04	32.5	58.3	162	79.9	116.2	6.0	0.0719
	K	1.46	1.67	1.85	2.05	33.0	57.5	155	82.0	118.9	6.1	0.0710
^{184}W	Exp	1.35(12)	1.54(9)	2.00(18)	2.45(51)	1.8(3)		24(3)	37.1(28)	70.6(51)	4.0(4)	
	TH	1.45	1.63	1.76	1.90	0.63	1.58	7.57	123.30	178.08	9.11	0.0701
	MR	1.49	1.78	2.07	2.43	24.36	55.19	220.78	108.02	160.65	8.71	0.0401
	M	1.48	1.71	1.88	2.04	30.5	57.4	167	124.8	182.0	9.4	0.0575
	D	1.48	1.73	1.95	2.16	40.7	75.2	216	128.3	187.3	9.8	0.0494
	K	1.48	1.73	1.97	2.27	38.9	71.8	214	128.4	187.1	9.8	0.0420
^{186}W	Exp	1.30(9)	1.69(12)	1.60(12)	1.36(36)				41.7(92)	91.0(201)		

Table 4: (continued)

nucl.		$\frac{4g \rightarrow 2g}{2g \rightarrow 0g}$	$\frac{6g \rightarrow 4g}{2g \rightarrow 0g}$	$\frac{8g \rightarrow 6g}{2g \rightarrow 0g}$	$\frac{10g \rightarrow 8g}{2g \rightarrow 0g}$	$\frac{2\beta \rightarrow 0g}{2g \rightarrow 0g} \times 10^3$	$\frac{2\beta \rightarrow 2g}{2g \rightarrow 0g} \times 10^3$	$\frac{2\beta \rightarrow 4g}{2g \rightarrow 0g} \times 10^3$	$\frac{2\gamma \rightarrow 0g}{2g \rightarrow 0g} \times 10^3$	$\frac{2\gamma \rightarrow 2g}{2g \rightarrow 0g} \times 10^3$	$\frac{2\gamma \rightarrow 4g}{2g \rightarrow 0g} \times 10^3$	rms
	TH	1.44	1.63	1.76	1.90	53.31	87.87	217.62	176.39	254.06	12.92	0.1025
	MR	1.39	1.49	1.52	1.52	726.29	1022.06	1780.46	168.58	240.37	11.94	0.0569
	M	1.49	1.76	1.99	2.20	31.4	64.7	213	164.0	241.2	12.7	0.1604
	D	1.51	1.80	2.07	2.34	46.2	91.9	289	165.7	244.5	12.9	0.1874
	K	1.49	1.77	2.08	2.48	47.3	89.1	275	174.0	254.4	13.3	0.2081
¹⁸⁶ Os	Exp	1.45(7)	1.99(7)	1.89(11)	2.06(44)				109.4(71)	254.6(150)	13.0(47)	
	TH	1.45	1.64	1.79	1.96	2.66	7.29	38.37	175.96	254.48	13.04	0.0559
	MR	1.49	1.77	2.05	2.40	24.18	54.05	212.38	141.94	209.52	11.18	0.0629
	M	1.50	1.80	2.11	2.45	26.2	59.5	235	163.5	242.2	13.0	0.0700
	D	1.53	1.87	2.20	2.55	39.7	90.2	335	164.9	247.4	13.4	0.0852
	K	1.50	1.81	2.16	2.63	39.2	81.0	288	173.5	255.9	13.6	0.0940
¹⁸⁸ Os	Exp	1.68(11)	1.75(11)	2.04(15)	2.38(32)				63.3(92)	202.5(304)	43.0(74)	
	TH	1.46	1.68	1.88	2.09	2.62	9.00	61.14	257.60	374.78	19.39	0.0691
	MR	1.51	1.84	2.22	2.69	25.17	62.78	283.13	228.55	339.59	18.25	0.0654
	M	1.51	1.84	2.22	2.73	23.0	56.7	257	245.3	363.5	19.4	0.0713
	D	1.54	1.89	2.25	2.63	33.9	83.9	344	229.8	345.2	18.7	0.0626
	K	1.52	1.87	2.29	2.87	33.6	78.5	330	246.6	366.2	19.7	0.0903
²³⁰ Th	Exp	1.36(8)				5.7(26)		20(11)	15.6(59)	28.1(100)	1.8(11)	
	TH	1.44	1.60	1.70	1.79	9.12	15.01	36.85	67.59	97.15	4.92	0.0190
	MR	1.45	1.64	1.79	1.93	18.38	33.20	94.91	64.73	93.67	4.82	0.0231
	M	1.47	1.69	1.88	2.07	23.6	45.3	141	62.5	91.4	4.8	0.0296
	D	1.47	1.70	1.90	2.09	30.0	56.4	168	63.6	93.2	4.9	0.0331
	K	1.46	1.68	1.86	2.07	31.4	55.6	155	66.9	97.3	5.1	0.0311
²³² Th	Exp	1.44(15)	1.65(14)	1.73(12)	1.82(15)	14(6)	2.6(13)	17(8)	14.6(28)	36.4(56)	0.7	
	TH	1.44	1.59	1.69	1.76	353.24	517.84	989.55	77.75	111.45	5.62	0.1159
	MR	1.45	1.63	1.76	1.88	16.73	29.21	78.75	60.22	86.84	4.43	0.0118
	M	1.46	1.66	1.82	1.98	21.3	39.1	115	56.7	82.4	4.3	0.0219
	D	1.46	1.67	1.84	2.01	25.8	47.1	135	57.0	83.0	4.3	0.0261
	K	1.45	1.65	1.80	1.96	26.0	44.9	119	61.9	89.6	4.6	0.0203
²³⁴ U	Exp								12.5(27)	21.1(44)	1.2(3)	
	TH	1.43	1.59	1.68	1.75	2.17	4.12	12.88	45.04	64.64	3.27	0.0181
	MR	1.44	1.62	1.74	1.85	15.82	27.15	71.03	43.90	63.30	3.23	0.0175
	M	1.45	1.63	1.77	1.89	18.5	32.4	88	42.8	62.0	3.2	0.0170
	D	1.45	1.64	1.78	1.90	20.7	36.1	97	42.7	61.8	3.2	0.0169
	K	1.45	1.65	1.80	1.96	26.0	44.9	119	61.9	89.6	4.6	0.0282
²³⁶ U	Exp	1.42(11)	1.55(11)	1.59(17)	1.46(17)							
	TH	1.43	1.59	1.68	1.75	3.32	6.31	19.53	44.47	63.82	3.22	0.0760
	MR	1.44	1.61	1.73	1.83	14.90	25.17	64.00	45.34	65.25	3.32	0.0996
	M	1.45	1.63	1.75	1.87	17.6	30.4	80	44.6	64.4	3.3	0.1115
	D	1.45	1.63	1.76	1.87	19.3	33.2	87	44.7	64.5	3.3	0.1124
	K	1.44	1.62	1.75	1.86	19.5	32.7	82	45.5	65.6	3.3	0.1086
²³⁸ U	Exp			1.45(23)	1.71(22)	1.4(6)	3.6(14)	12(5)	10.8(8)	18.9(17)	1.2(1)	
	TH	1.43	1.59	1.68	1.74	3.21	6.00	18.11	38.10	54.67	2.76	0.0292
	MR	1.44	1.62	1.73	1.83	15.23	25.88	66.47	38.61	55.65	2.84	0.0389
	M	1.45	1.62	1.75	1.86	17.2	29.6	77	37.7	54.4	2.8	0.0429
	D	1.45	1.63	1.75	1.86	18.9	32.3	83	37.7	54.5	2.8	0.0430
	K	1.44	1.62	1.74	1.85	19.3	32.3	80	39.4	56.8	2.9	0.0414
²³⁸ Pu	Exp					14(4)		11(4)				
	TH	1.43	1.59	1.67	1.74	1.50	2.75	8.10	41.80	59.92	3.02	0.0067
	MR	1.44	1.61	1.73	1.83	14.88	25.13	63.88	41.24	59.38	3.02	0.0262
	M	1.44	1.60	1.70	1.78	6.1	10.0	24	42.4	60.8	3.1	0.0075
	D	1.44	1.62	1.73	1.84	19.1	31.7	78	41.6	59.9	3.0	0.0334
	K	1.44	1.61	1.73	1.82	18.8	30.8	74	42.2	60.7	3.1	0.0314
²⁵⁰ Cf	Exp								6.8(17)	10.9(25)	0.6(1)	
	TH	1.43	1.59	1.68	1.75	3.29	6.26	19.43	38.82	55.74	2.82	0.0184

Table 4: (continued)

nucl.	$\frac{4g \rightarrow 2g}{2g \rightarrow 0g}$	$\frac{6g \rightarrow 4g}{2g \rightarrow 0g}$	$\frac{8g \rightarrow 6g}{2g \rightarrow 0g}$	$\frac{10g \rightarrow 8g}{2g \rightarrow 0g}$	$\frac{2\beta \rightarrow 0g}{2g \rightarrow 0g} \times 10^3$	$\frac{2\beta \rightarrow 2g}{2g \rightarrow 0g} \times 10^3$	$\frac{2\beta \rightarrow 4g}{2g \rightarrow 0g} \times 10^3$	$\frac{2\gamma \rightarrow 0g}{2g \rightarrow 0g} \times 10^3$	$\frac{2\gamma \rightarrow 2g}{2g \rightarrow 0g} \times 10^3$	$\frac{2\gamma \rightarrow 4g}{2g \rightarrow 0g} \times 10^3$	rms
MR	1.44	1.61	1.72	1.81	14.13	23.58	58.63	39.83	57.29	2.91	0.0190
M	1.44	1.60	1.71	1.80	13.0	21.6	53	40.1	57.7	2.9	0.0192
D	1.44	1.61	1.72	1.81	15.0	24.9	61	40.0	57.5	2.9	0.0191

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